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Bambang Piscesa
University of New South Wales

Mario M. Attard
University of New South Wales

Ali Khajeh Samani
University of New South Wales

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REFINED PLASTICITY MODEL FOR CONCRETE STRESS-STRAIN RELATIONSHIP PART II: INCLUSION OF SIZE EFFECT

Bambang Piscesa
School of Civil Engineering and Environment, The University of New South Wales
Sydney, NSW 2052, Australia. bambang.piscesa@unsw.edu.au

Mario M. Attard*
School of Civil Engineering and Environment, The University of New South Wales
Sydney, NSW 2052, Australia. m.attard@unsw.edu.au (Corresponding Author)

Ali Khajeh Samani
School of Civil Engineering and Environment, The University of New South Wales
Sydney, NSW 2052, Australia. ali.samani@unsw.edu.au

ABSTRACT

This article will discuss about the implementation of size effect into the proposed plasticity model by adjusting the plastic potential function or flow rule. A new parameter (α_{P0}) that control the plastic dilation rate after the peak is proposed. In addition a method to include the initial plastic compacting during the plastic flow is also proposed and discussed in detail. The size effect is implemented by means of calibration the softening function in both axial direction and lateral direction with specific aspect ratio of height by width is equal to 2. Some comparison of the proposed plasticity model including the proposed plastic potential function with experimental results of several researcher is compared and discussed in detail.

KEYWORDS

Plasticity model, concrete stress-strain, triaxial-compression, plastic potential, size effect, flow rule.

INTRODUCTION

A plastic potential in which have a plastic dilation control after the peak is proposed. This plastic potential is an improvement of the original model (Grassl, Lundgren et al. 2002) where in this paper only compression region is considered. Two research significance by using the proposed plastic potential is discussed. First, the plastic dilation rate after the peak can be controlled to simulate the localization due to tensile splitting cracking and also a shear band (Markeset and Hillerborg 1995). Second, the plastic potential have an initial plastic compaction when the plastic flow is started in which this kind of behaviour cannot be acquired as the results of plastic flow always positive in the original model. One additional parameter that controlling the plastic dilation rate after the peak (α_{P0}) is introduced and a method to include the initial plastic compaction is discussed in this paper. Several comparison of experimental results with the proposed model in triaxial compression both in axial and lateral component is shown. In addition the capability of proposed plasticity model to incorporate the size effect is also compared with experimental results.

PLASTIC POTENTIAL

Non-associated flow rule (Grassl, Lundgren et al. 2002) is modified by adding additional parameter to control the plastic dilation rate after the peak has a general form as follows:

$$g = -A\left(\frac{\rho}{\sqrt{q(k)}}\right)^2 - B\frac{\rho}{\sqrt{q(k)}} + \alpha_{P0}\frac{\xi}{\sqrt{q(k)}} = 0 \quad (1)$$

Where the coefficient A and B need to be calibrated with the axial strain at maximum peak stress in uniaxial and in a triaxial compression. To obtain the basic coefficient of A and B are discussed in detail by (Grassl, Lundgren et al. 2002) and shown in here is the final equation as follows:

$$A = \frac{\psi_2 - \psi_1}{2(\rho_1 - \rho_2)}\ ,\ B = \rho_1\frac{\psi_1 - \psi_2}{(\rho_1 - \rho_2)} - \psi_1 \quad (2)$$

Where ρ_1 and ρ_2 are the normalized length of deviatoric vector in uniaxial and triaxial state. ψ_1 and ψ_2 are inclination of plastic potential for uniaxial and triaxial state. The expression for those parameter are

$$\rho_1 = \sqrt{\frac{2}{3}}\ ,\ \rho_2 = \sqrt{\frac{2}{3}}\left(\frac{\sigma_{1cc} - \sigma_{3cc}}{f_c^{'}}\right)\ ,\ \psi_1 = -\sqrt{2}\frac{\varepsilon_{3pu} - \varepsilon_{1pu}}{\varepsilon_{v,\text{peak}}^{p}}\ ,\ \psi_2 = -\sqrt{2}\frac{\varepsilon_{3pc} - \varepsilon_{1pc}}{\varepsilon_{v,\text{peak}}^{p}} \quad (3)$$

Where σ_{1cc} and σ_{3cc}, are the lateral stress and axial peak stress at triaxial state. ε_{3pu} and ε_{1pu} are axial plastic strain and lateral plastic strain at uniaxial state. ε_{3pc} and ε_{1pc} are axial plastic strain and lateral plastic strain at triaxial state.

DILATION CHARACTERISTIC AND IMPLEMENTATION OF SIZE EFFECT ON LATERAL COMPONENT

The implementation of size effect is treated as the parameter that control the plastic dilation rate of concrete. This parameter can be evaluated by deriving the plastic potential with its hydrostatic part $\partial g / \partial \xi$, in which has been introduced by (Han and Chen 1985). The first derivative of plastic potential function with its hydrostatic part are as follows

$$\frac{\partial g}{\partial \xi} = \alpha_{P0}\frac{1}{\sqrt{q(k)}} \quad (4)$$

The parameter α_{P0} will only be applied after the peak as the formulation of plastic dilation rate used herein. This parameter is measured as a plastic dilation rate from the peak (Samani and Attard 2014). This assumption led the value of α_{P0} before the peak equal to unity. To ensure the continuity of plastic dilation rate in stress-strain relationship before the peak and after the peak the value of A and B must be updated. At its peak with the corresponding value of A and B will give the value of α_{P0} equal to 1.
The formulation for plastic dilation rate (β_{size}) after the peak is taken from (Samani and Attard 2014) where it was calibrated with experimental results that has different size of specimen. The general expression for β_{size} is rewritten here as:

$$\beta_{size} = \frac{c}{a\left(\frac{f_r}{f_c^{'}}\right)^b + 1} - 0.5 \quad (5)$$

Parameters a, b and c can be expressed as

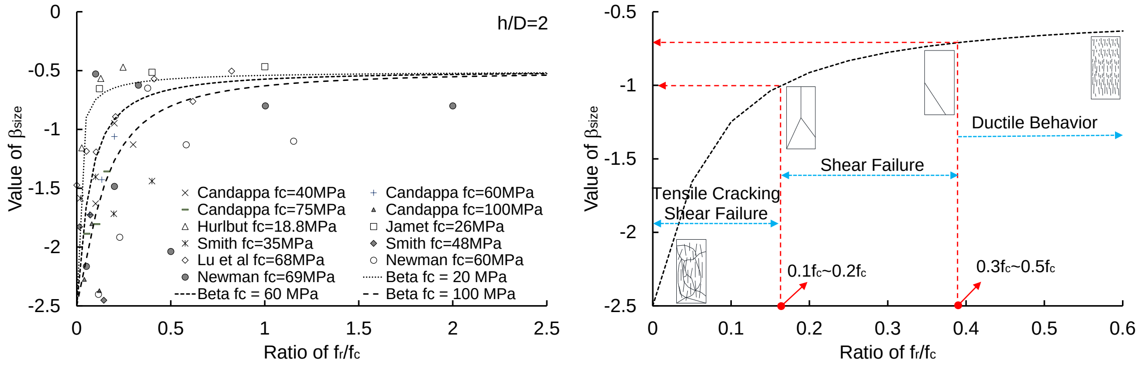
$$a = 65e^{-0.015 f_c^{'}}\ ,\ b = 1.5 - e^{-0.02 f_c^{'}}\ ,\ c = -4\left(\frac{D}{h}\right) \text{ for } h \le h_d\ ,\ c = -2 \text{ for } h > h_d \quad (6)$$

Where e, D, h ,h_d, a, b and c are the natural base of logarithmic, diameter of specimen, height of specimen, height damage zone in specimen and the rest are parameter in plastic dilation rate function respectively. The relationship of β_{size} with experimental results is shown in Figure 1. Figure 1 shows that β_{size} is affected by the concrete compressive stress and also confining stress. This β_{size} is also show the type of failure where at low confinement the failure is dominated by tensile cracking and shear failure, as the confinement increasing the tensile cracking is nullified and led to shear failure. When high confining pressure applied the ductile behavior of concrete is reached.

In order to implement the β_{size} into the flow rule after the peak we can consider the relation between the lateral plastic strain with axial strain as follow

$$\beta_{size} = \frac{d\varepsilon_1^p}{d\varepsilon_3^p} = \frac{d\lambda \dot{g}_1}{d\lambda \dot{g}_3} = \frac{\dot{g}_1}{\dot{g}_3} \tag{7}$$

Where $d\varepsilon_1^p$ and $d\varepsilon_3^p$ are the lateral plastic strain increment and axial plastic strain increment, respectively.



(a) (b)

Figure 1. Comparison of β_{size} with experimental results (a) and failure behavior related to β_{size} (b)

The derivation of plastic potential with corresponding stress are

$$\dot{g}_1 = -\frac{2A(\sigma_1-\sigma_3)}{3f_c^2 q(k)} - \frac{B}{\sqrt{6}f_c\sqrt{q(k)}} + \alpha_{P0}\frac{(1)}{f_c\sqrt{3q(k)}},\ \dot{g}_3 = \frac{4A(\sigma_1-\sigma_3)}{3f_c^2 q(k)} + \frac{2B}{\sqrt{6}f_c\sqrt{q(k)}} + \alpha_{P0}\frac{(1)}{f_c\sqrt{3q(k)}} \tag{8}$$

By using the value of k at specific stress after the peak is reached and using the β_{size} value derived from Eq. (5) we can calculate the α_{P0}. Thus the value of ε_1^P and ε_3^P will need to be calculated using the new value of α_{P0}. The general expression of solved α_{P0} using Eq.(7)~(8) are

$$\alpha_{P0} = \left[(2+4\beta_{size})\left(\frac{A(\sigma_1-\sigma_3)}{3f_c^2 q(k)}\right) + (1+2\beta_{size})\left(\frac{B}{\sqrt{6}f_c\sqrt{q(k)}}\right)\right]\left[(1-\beta_{size})\frac{(1)}{f_c\sqrt{3q(k)}}\right]^{-1} \tag{9}$$

To ensure the continuity of plastic potential function as mentioned before the value of A and B needed to be updated. The assumption needed to satisfy is the total volumetric strain at peak is equal to zero ($\varepsilon_{v,peak}^{P} + \varepsilon_{v,peak}^{e} = 0$). To achieve that the initial peak plastic dilation rate at uniaxial state μ_u and triaxial state μ_c state must be derived first. The detail for deriving the peak plastic dilation at uniaxial are as follows,

$$\varepsilon_{v,\text{peak},u}^{e} = 2\varepsilon_{1eu} + \varepsilon_{3eu},\ \varepsilon_{v,\text{peak},u}^{p} = 2\varepsilon_{1pu} + \varepsilon_{3pu} \tag{10}$$

Where ε_{1eu} and ε_{1eu} are lateral elastic strain and axial elastic strain at peak, respectively. Substituting Eq.(10) into the condition at peak ($\varepsilon_{v,peak}^{P} + \varepsilon_{v,peak}^{e} = 0$) will gives,

$$2\varepsilon_{1pu} + \varepsilon_{3pu} + \varepsilon_{v,\text{peak},u}^{e} = 0 \tag{11}$$

Using the relation of peak plastic dilation at uniaxial state, $\mu_u=\varepsilon_{1pu}/\varepsilon_{3pu}$ into Eq. (11) lead to:

$$2(\mu_u\varepsilon_{3pu}) + \varepsilon_{3pu} + \varepsilon_{v,\text{peak},u}^{e} = 0 \rightarrow \mu_u = \frac{-\varepsilon_{v,\text{peak},u}^{e} - \varepsilon_{3pu}}{2\varepsilon_{3pu}} \tag{12}$$

By using the same method above the peak plastic dilation at triaxial state can be deployed as follows,

$$\mu_c = \frac{-\varepsilon_{v,\text{peak},c}^{e} - \varepsilon_{3pc}}{2\varepsilon_{3pc}} \tag{13}$$

After Eq.(12) and Eq. (13) is calculated then at each incremental plastic volumetric strain the inclination of plastic potential at uniaxial state ψ_1 and triaxial state ψ_2 must be updated. The detail formulation for dynamic updating of plastic potential inclination in uniaxial and triaxial state are as follows,

$$n_c = \left(\frac{\beta_{size}^{c}}{2\mu_c}\right)^2,\ n_u = \left(\frac{\beta_{size}^{u}}{2\mu_u}\right)^2 \tag{14}$$

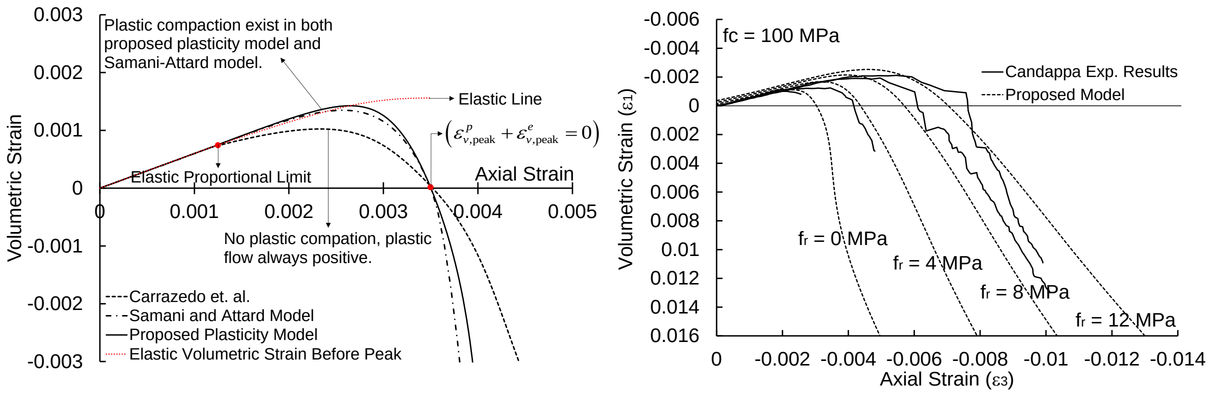
$$\beta_{size}^{c,k} = (k)^{n_c}\,\beta_{size}^{c}, \beta_{size}^{u,k} = (k)^{n_u}\,\beta_{size}^{u} \tag{15}$$

Where n_c, n_u, $\beta_{size}^{c,k}$, $\beta_{size}^{u,k}$, β_{size}^{c} and β_{size}^{u} are rate of changing in dilation ratio in triaxial and uniaxial, corresponding plastic dilation ratio with specific plastic volumetric incremental in triaxial and uniaxial, plastic dilation ratio of concrete at peak in which includes the size effects in triaxial and uniaxial, respectively. By knowing the relation of $\beta_{size}^{c,k} = \varepsilon_{1pc}^{k}\left(\varepsilon_{3pc}\right)^{-1}$ and also $\beta_{size}^{u,k} = \varepsilon_{1pc}^{k}\left(\varepsilon_{3pc}\right)^{-1}$ then further implementation would be,

$$\varepsilon_{v,k}^{p} = 2\varepsilon_{1pc}^{k} + \varepsilon_{3pc} = 2\beta_{size}^{c,k}\varepsilon_{3pc} + \varepsilon_{3pc} = \left(2\beta_{size}^{c,k}+1\right)\varepsilon_{3pc} \tag{16}$$

$$\psi_1 = -\sqrt{2}\,\frac{\varepsilon_{3pu}-\varepsilon_{1pu}^{k}}{\varepsilon_{v,\mathrm{peak}}^{p}} = -\sqrt{2}\,\frac{\varepsilon_{3pu}-\varepsilon_{3pu}\beta_{size}^{u,k}}{\varepsilon_{v,\mathrm{peak}}^{p}}\ ,\ \psi_2 = -\sqrt{2}\,\frac{\varepsilon_{3pc}-\varepsilon_{1pc}^{k}}{\varepsilon_{v,\mathrm{peak}}^{p}} = -\sqrt{2}\,\frac{\varepsilon_{3pc}-\varepsilon_{3pc}\beta_{size}^{c,k}}{\varepsilon_{v,\mathrm{peak}}^{p}} \tag{17}$$

Where ε_{1pc}^{k} and ε_{1pu}^{k} are the lateral strain at specific plastic volumetric incremental in triaxial and uniaxial, respectively. When calculating the $\beta_{size}^{u,k}$ the confinement pressure (f_r) can be set to zero to conform with uniaxial state and consequently the $\beta_{size}^{c,k}$ will equal to β_{size} with specific value of confinement pressure (f_r). After the peak strain is reached the value of $\varepsilon_{v,\mathrm{peak}}^{p}$ will be equal to $\varepsilon_{v,\mathrm{peak,c}}^{p}$ and no further changing in parameter A and B. The comparison of dilation characteristic with other researcher, (e.g. Samani and Attard 2014;Carrazedo, Mirmiran et al. 2013) and experimental results (Candappa, Sanjayan et al. 2001) is shown in Figure 2.



(a) (b)

Figure 2. Dilation characteristic of proposed model compared with other researcher

IMPLEMENTATION OF SIZE EFFECT IN AXIAL COMPONENT

Implementation of size effect in axial component is simply consist of calibration of proposed model with experimental result that is used as a reference and also with different aspect ratio. Here aspect ratio of h/2 = 2 is used as a reference. A predefined control point from experimental results at half point between peak (f_{cc}) and residual (f_{res}) are selected and the same point in stress-strain model should pass the control point (f_x=f_{res}+0.5(f_{cc}-f_{res})). When this condition met the specific value of t in softening function is then can be said calibrated.

Additional adjustment are made on the resulting value of t derived from experimental results where the density of concrete in each experimental results is differs. It is then normalized into one value for general purpose. This value corresponds with normal density of concrete (ρ_{con} = 2400 kg/m^3). The relationship of t value with various density in respect with normal density of concrete is obtained by conducting parametric study in which consist of comparison between recent empirical model (Samani and Attard 2012) and the proposed plasticity model. The results of this parametric study shown that as the concrete density is lower than specified the higher value of t is obtained and vice versa.

The value of t in which obtained by calibrating with experimental results has different value depending on confinement pressure. It is found that for low concrete strength the value of t is somehow become lower after confinement pressure is applied and for high concrete strength the value of t is become higher after confinement pressure is applied. Based on that phenomenon and also to simplify selection for value of t for general purpose the proposed equation for concrete material can be expressed as

$$t_{ref} = \frac{62.159}{1000} 1.35^{\left(\frac{2400-\rho_{con}}{400}\right)} \left(f_c^{'}\right)^{-0.501} \text{ for } \frac{f_r}{f_c} = 0 \tag{18}$$

$$t_{ref} = \frac{1.845}{1000} 1.35^{\left(\frac{2400-\rho_{con}}{400}\right)} \left(f_c^{'}\right)^{0.4236} \text{ for } \frac{f_r}{f_c} \geq 0.01 \tag{19}$$

If a very small value of f_r/f_c is used, for example $0 < f_r/f_c < 0.01$ then linear interpolation from Eq.(18) and Eq.(19) can be used.

The calibration for axial strain behavior due to size effect is based on experimental results (e.g. Vonk 1993; Nakamura and Higai 2001; Jansen and Shah 1997). The calibration was conducted using targeted value of t in order to eliminate the difference issues of experimental results with different researcher. Since there is no complete experimental results that varied from h/w = 0.5 to h/2 = 5.5 then the results (e.g Vonk 1993; Jansen and Shah 1997) with the same concrete compressive strength is used. The proposed value of t to incorporate the size effect has a form of equation as follows

$$t_{size} = t_{ref} \left(\frac{h}{2w}\right)^{-1.454} \quad for\ h \leq h_d\ ,\ t_{size} = t_{ref} \left(\frac{h}{2w}\right)^{-0.871} \quad for\ h > h_d \tag{20}$$

Where h_d is the length of damage zone in which represent the localization height that consist of tensile cracking and shear band. When calculating the softening parameter in Eq.(18)~(19), the value of t should be updated using Eq.(20). The prediction of t value in estimating the size effect using Eq.(20) is shown in Figure 4.

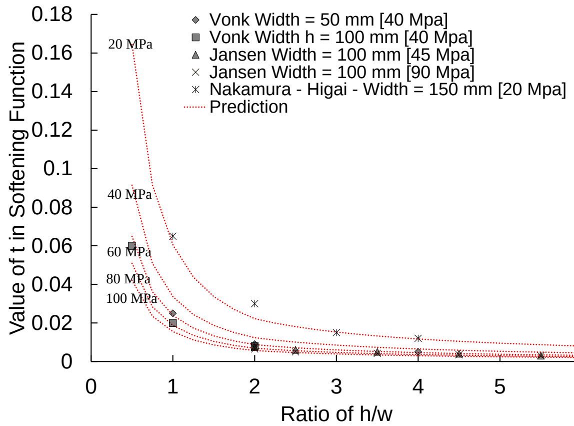


Figure 3. Proposed value of t compared with the value derived from experimental results

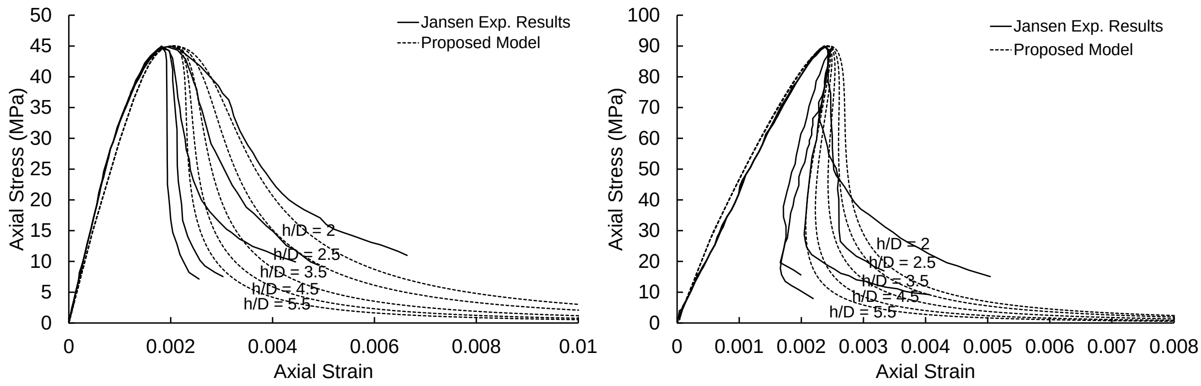
COMPARISON WITH EXPERIMENTAL RESULTS

Comparison with experimental results that represent the size effect by (Jansen and Shah 1997) is shown in Figure 4. Input parameter used with the proposed model is shown in Table 1 (The value of t in Table 1 is according to Eq.(20)). The implementation of size effect in the proposed model has been successful to capture the behavior of concrete in compression under different aspect ratio as shown in Figure 4. In addition to that the proposed model also able to capture the snapback behavior of stress-strain curve in the softening branch.

Table 1. Input General Model Parameter in Comparison with Experimental Results

Researcher	f_c (MPa)	ρ (kg/m^3)	E_c (MPa)	f_r (MPa)	E_a	$t_{uniaxial}$,$t_{triaxial}$	d, h (mm)	Silica Fume
Jansen NSC	45.00	2300	31817	0	4	0.00995,- 0.00819,- 0.00611,- 0.00491,- 0.00412,-	100,200 100,250 100,350 100,450 100,550	No

Researcher	f_c (MPa)	ρ (kg/m^3)	E_c (MPa)	f_r (MPa)	E_a	$t_{uniaxial}$,$t_{triaxial}$	d, h (mm)	Silica Fume
Jansen HSC	90.00	2800	50908	0	1.5	0.004832,- 0.003978,- 0.002968,- 0.002384,- 0.002002,-	100,200 100,250 100,350 100,450 100,550	Yes



(a) fc = 45 MPa (b) 90 MPa

Figure 4. Comparison of proposed model with (Jansen and Shah 1997) experimental results.

CONCLUSION

In this part a plastic potential function that has a new parameter (α_{P0}) that control the dilation rate is proposed. The inclusion of size effect is being done by calibrating the value of t in softening parameter with experimental results that have a different aspect ratio. A method that update the value of A and B in proposed plastic potential is successful in modelling the initial plastic compaction during the plastic flow. Comparison with experimental results shown a very good agreement in both axial component and lateral component of stress strain model.

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