23rd Australasian Conference on the Mechanics of Structures and Materials

2014

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Publication details

Visintin, P & Oehlers, DJ 2014, 'Mechanics solutions for quantifying ductility and moment redistribution at all load levels', in ST Smith (ed.), *23rd Australasian Conference on the Mechanics of Structures and Materials (ACMSM23)*, vol. I, Byron Bay, NSW, 9-12 December, Southern Cross University, Lismore, NSW, pp. 301-306. ISBN: 9780994152008.

23rd Australasian Conference on the Mechanics of Structures and Materials (ACMSM23)
Byron Bay, Australia, 9-12 December 2014, S.T. Smith (Ed.)

MECHANICS SOLUTIONS FOR QUANTIFYING DUCTILITY AND MOMENT REDISTRIBUTION AT ALL LOAD LEVELS

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ABSTRACT

For the efficient design of reinforced concrete beams and frames, moment redistribution is used to: reduce the absolute maximum magnitude of the moment in the critical region; equalise the critical moments at either side of interior columns and fully utilise the capacity of the non-critical regions of a member. Although important in design, moment redistribution has historically proved to be difficult to quantify due to the complexity of quantifying hinge rotations. In this paper a framework used to develop mechanics based equations to quantify the moment rotation characteristics of reinforced concrete hinges is presented. This framework directly applies partial interaction theory in the tension region to simulate the mechanisms associated with slip of the reinforcement relative to the surrounding concrete as cracks widen. While in the compression region, shear friction theory is used to describe the formation and failure of concrete softening wedges. Mechanics based solutions for moment redistribution at all load levels is presented and it is shown that at the ultimate limit state mechanics based closed form solutions simple enough for use in routine design can be developed.

KEYWORDS

Ductility, moment redistribution, analytical solution, partial interaction, shear friction.

INTRODUCTION

Moment redistribution is vital to the design of reinforced concrete (RC) structures as it governs the strength of continuous members and the ability to absorb energy such as that from dynamic events. Moreover moment redistribution is used to improve the efficiency of design by allowing for a reduction in the absolute magnitude of moments in the critical regions; the equalisation of moments at either side of interior columns; and the full utilisation of the capacity of non-critical regions of a member (Paulay 1976).

Despite the importance of moment redistribution predicting the moment redistribution capacity has historically proved difficult. While numerous empirically derived hinge lengths, that is the length over which the full interaction curvature at failure is integrated to give rotations, have been proposed, when applied to a global dataset correlation is poor (Panagiatakos and Fardis 2001). The authors (Visintin et al. 2012; Oehlers et al. 2014a; Oehlers et al. 2014b) have suggested that the difficulty in predicting the ductility of RC members lies in the method of analysis most often applied and suggested by codes, that is in the application of the full interaction moment curvature approach (M/χ). This is because at all load levels following the formation of an initial crack the moment M/χ approach does not simulate the localised partial interaction behaviours seen in practice. These partial interaction behaviours are: (1)

the slip of the reinforcement relative to the surrounding concrete in the tension region, which controls crack formation, crack widening and tension stiffening; and (2) the slip along concrete to concrete sliding planes in the compression region which are associated with the formation and failure of concrete softening wedges.

In response to the limitations imposed by empiricisms, in this paper a segmental moment rotation approach developed by the authors (Visintin et al. 2012; Oehlers et al. 2014a; Oehlers et al. 2014b; Visintin and Oehlers 2014a) is presented. The segmental approach is based on the Euler-Bernoulli theorem of plane section but not on the corollary of a linear strain profile. The approach directly simulates the formation and widening of cracks as well as tension stiffening through the application of partial interaction theory and the formation and failure of concrete softening wedges through the application of shear friction theory in the form of a size dependent stress strain relationship.

In this paper the partial interaction mechanisms which control the ductility of RC members are first described, a segmental approach which directly incorporated these mechanisms is then presented. Finally, mechanics solutions to define moment redistribution and which incorporate the segmental approach and therefore directly simulate the partial interaction mechanisms are given.

PARTIAL INTERACTION MECHANISMS

Tension Stiffening Mechanism

Consider an RC prism such as that shown in Figure 1(a) which is extracted from the tension region of the beam in Figure 2. The concrete prism has a cross sectional area of A_c and modulus E_c and is reinforced with a total area of reinforcement A_r and modulus E_r. The prism is uniformly stressed such that prior to cracking it has a total axial rigidity of $E_rA_r+E_cA_c$. After the formation of an initial crack the axial rigidity of the prism reduces but due to tension stiffening is greater than that of the reinforcement alone. This increase in stiffness above that of the bare bar can be directly simulated through mechanics by considering the load slip P_{rt}/Δ_{rt} behavior of the prism, the full details of which can be found elsewhere (Haskett et al. 2008;Visintin et al. 2012), but a summary of which is presented here.

After an initial crack has developed the total force in the prism in Figure 1(a) is resisted by the reinforcement force at the crack face, $P_{rt,}$ in Figure 1(b). The force P_{rt} induces an interface slip δ along the length of the bar and is at its maximum Δ_{rt} at the crack face and reduces to zero at a distance S_{cr}. The distribution of slip δ in Figure 1(d) is itself a function of the interface shear stress distribution in Figure 1(c) which can be defined from local bond stress slip (τ/δ) material models. From the known distribution of shear stress and slip, the distribution of strain in the reinforcement ε_r and in the concrete ε_c can be determined. The slip strain $d\delta/dx$ shown in Figure 1(e), which is defined as the difference in strain between the reinforcement and the concrete can therefore be determined. Knowing the distribution of δ and $d\delta/dx$ the full interaction boundary condition can be identified as the location where both δ $d\delta/dx$ reduce to zero. Beyond the full interaction boundary condition the stress in the reinforcement and the concrete are constant and the stress in the concrete is at its maximum. Hence a subsequent crack can form anywhere in the full interaction region but the minimum primary crack spacing is S_{cr}.

When primary cracks have formed along the prism as in Figure 1(f) the loading on each prism is symmetric and hence so too is the distribution of slip and shear-stress as shown in Figures 1(g) and 1(h). Through symmetry, the boundary condition that the slip at the midpoint between two cracks is zero can be applied to determine the relationship between P_{rt} and Δ_{rt}. Hence, through a PI analysis of the prism in Figure 1(a), the crack spacing S_{cr} can be determined and through the PI analysis in Figure 1(f), relationship between P_{rt} and Δ_{rt}, which allows for tension stiffening through bond slip is known. It should be noted that full details of a numerical solution procedure (Haskett et al. 2008;Visintin et al. 2012) as well as closed form solutions (Zhang et al. 2014) which define both the crack spacing and the stiffness of the P_{rt}/Δ_{rt} can be found elsewhere.

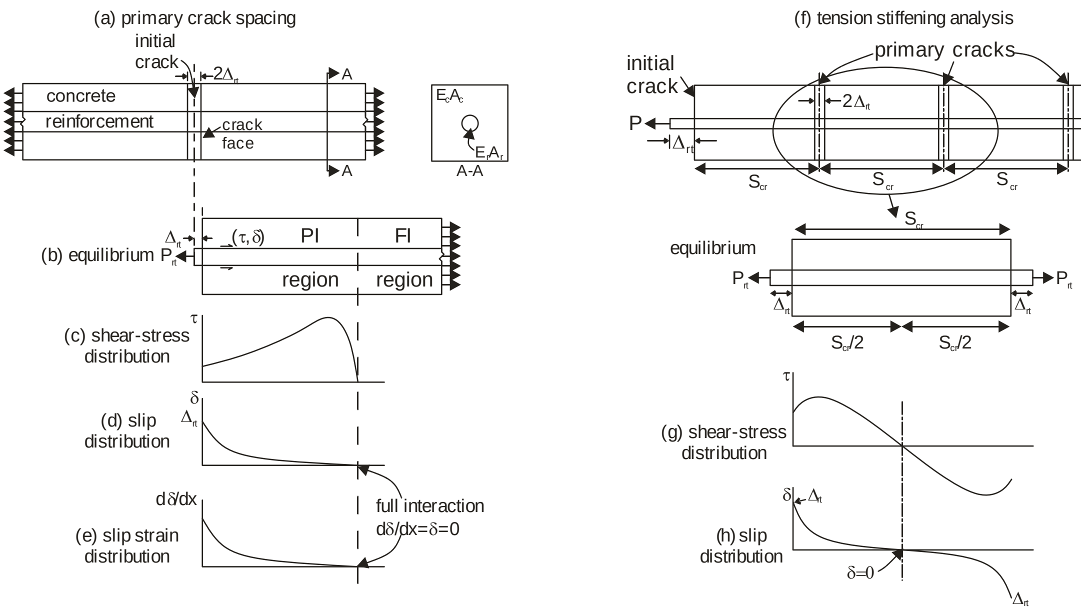

Figure 1. Partial interaction tension stiffening analysis

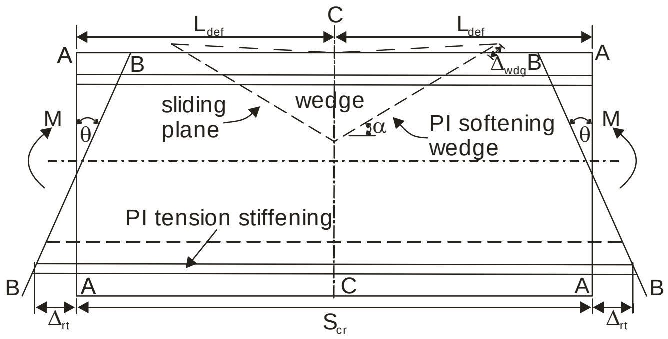

Figure 2. Segment of an RC beam

Shear Friction Mechanism for Concrete Softening

Let us now consider the partial interaction mechanism which can be used the formation and failure of the concrete softening wedge in Figure 2 (Oehlers et al. 2014a; Visintin et al. 2014c). The shear friction mechanism of concrete softening can be defined by considering a prism of total length $2L_{def}$ = S_{cr} which is subjected to a concentric load σ_a in Figure 3(a) (Chen et al. 2014; Oehlers et al. 2014a).

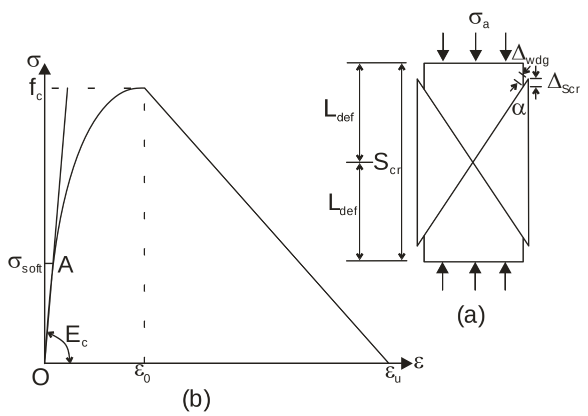

Figure 3. Shear friction concrete softening mechanism

On gradual loading of the prism in Figure 3(a), the stress strain behaviour O-A in Figure 3(b) is governed by material contraction according to the material modulus E_c. Beyond a stress σ_{soft} localised partial interaction sliding along potential planes of weakness at angle α result in the deformation Δ_{wdg} in Figure 3(a). For stresses above σ_{soft} the total shortening of the prism is therefore comprised of a material contraction $\sigma_a S_{cr}/E_c$ and a localised deformation $2\Delta_{Scr}$ where Δ_{Scr} is the longitudinal component of Δ_{wdg}. The total effective strain for a given stress greater than σ_{soft} is therefore σ_a/E_c+ $2\Delta_{Scr}/S_{cr}$. Importantly, the shear friction component, $2\Delta_{Scr}/S_{cr}$, is a pseudo strain and is size dependent. This is because for a given level of lateral confinement and axial stress, Δ_{Scr} remains constant regardless of size. It therefore follows that if the prism length S_{cr} in Figure 3 is taken as the length of the segment S_{cr} of in Figure 2 concrete softening will be simulated through the application of a size dependent stress-strain relationship such as by Chen et al. (2014) which has been determined using the concepts of shear friction as described here.

SEGMENTAL APPROACH

Having now defined the partial interaction mechanisms required for analysis let us consider their incorporation into a flexural analysis technique developed by the authors (Visintin et al. 2012; Visintin and Oehlers 2014a). Consider the segment in Figure 2(a) which is subjected to a constant moment M which causes a rotation at the ends θ which is accommodated by a deformation from A-A to B-B. This deformation is accommodated by a combination of material strains within the segment as well as the localised partial interaction mechanisms outlined above. For analysis, due to symmetry about plane C-C in Figure 2 only half the segment as shown in Figure 4(a) needs to be considered.

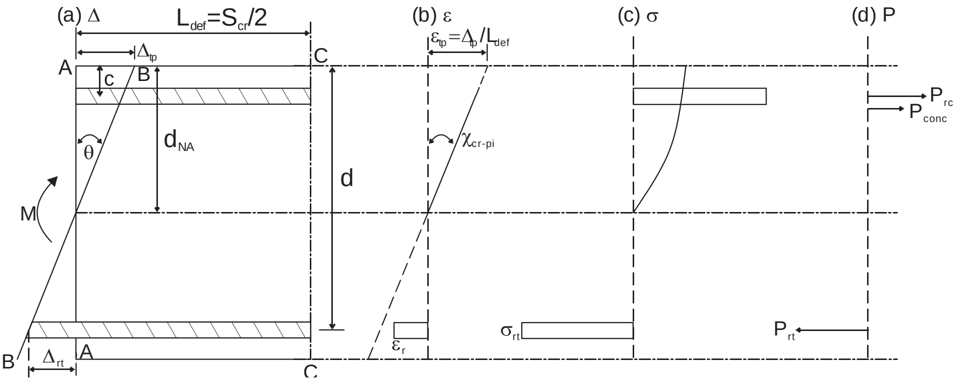

Figure 4. Segment for analysis

In Figure 4(a) if a rotation θ is imposed resulting in a deformation from A-A to B-B, the resulting strain profile in Figure 4(b) can be quantified by dividing the deformation from A-A to B-B by the length over which it must be accommodated, that is $L_{def} = S_{cr}/2$. Prior to any strain localisations, that is, either concrete cracking or softening, the distribution of stress in Figure 4(c) and hence internal forces in Figure 3(d) can be determined from the distribution of strain in Figure 4(b) through the application of material constitutive relationships. The neutral axis depth can then be adjusted until for a given rotation, equilibrium is achieved and hence the corresponding moment is known. Importantly it can be noted that prior to cracking this analysis is identical to the moment curvature approach which is mechanically correct.

Now consider the discontinuity in strain associated with concrete cracking and the slip of the reinforcement relative to the concrete which encases it. If the deformation from A-A to B-B in Figure 4(a) results in a slip at the crack face Δ_{rt} it is required that the corresponding force P_{rt} be known; this can be found using established partial interaction theory as described above. The analysis is therefore no longer a function of the linear strain profile in Figure 4(b), but rather a function of the linear displacement profile in Figure 4(a). That is the force in the reinforcement is directly determined from the displacement profile. Similarly when the rotation is increased such that concrete softening, which is characterised by the formation of a softening wedge occurs the application of shear friction theory through the use of a size dependent stress strain relationship is used to simulate concrete softening.

The above analysis can be used to derive a relationship between segment rotation and moment, importantly, the rotation θ can be divided by the half length of the segment $L_{def} = S_{cr}/2$ in Figure 4(a) to obtain an equivalent curvature, from which an equivalent cracked flexural rigidity can be obtained. Significantly, the equivalent curvature and flexural rigidity allow for tension stiffening and concrete softening through the use of mechanics rather than relying on empirical definitions.

Complete details of the segmental approach are presented in (Visintin et al 2012), closed form solutions to define the M/θ relationship at all load levels can be found in (Visintin and Oehlers 2014a) and effective flexural rigidities derived from the segmental approach for encastre beams in (Visintin and Oehlers 2014b).

MOMENT REDISTRIBUTION

According to Haskett et al. (2010) and Visintin et al. (2014c) let us define the moment redistribution factor K_{MR} for an encastr è beam as

$$K_{MR} = \frac{1}{1 + \frac{M}{\theta}\frac{L}{2EI}} \tag{1}$$

Where θ is the rotation of the hinge region at moment M and EI is the effective flexural rigidity of the span L.

From the segmental approach, closed form solutions for the M/θ characteristics of the hinge in Figure 4 are known (Visintin and Oehlers 2014a) at all load levels, as too are the effective flexural rigidities EI of an encastr è beam (Visintin and Oehlers 2014b). While the M/θ relationship derived from the closed form solutions can be applied to quantify moment redistribution at all load levels, for design it is required that the maximum possible amount of moment redistribution be determined.

According to (Visintin and Oehlers 2014a) if the ultimate rotation θ_{Tu} of a hinge can be defined as the rotation at which the maximum strain in the concrete is equal to the ultimate strain ε_u in Figure 3(b), and hence the ultimate hinge rotation $\theta_{Tu} = \varepsilon_u/\tan\alpha$, where α is the angle at which the softening wedge forms.

Applying the segmental approach outlined above and taking the tensile reinforcement to be yielded and perfectly plastic the moment corresponding the ultimate hinge rotation is

$$M_{\theta Tu} = P_{yt}d - \frac{(4\varepsilon_0 - \varepsilon_u)(P_{yt}d_{NA} + EA_{rc}\varepsilon_u(c - d_{NA}) - EA_{rc}c\varepsilon_u)}{12\varepsilon_0 - 4\varepsilon_u} + \frac{EA_{rc}c\varepsilon_u(c - d_{NA})}{d_{NA}} \tag{2}$$

Where P_{yt} is the yield force of the tensile reinforcement, ε_0 is the strain corresponding to the peak stress f_c, c is the cover to the centre of the compression reinforcement which has a axial rigidity of EA_{rc} and the neural axis depth d_{NA} is defined as

$$d_{NA} = -\frac{\varepsilon_u\left(3P_{yt} + \sqrt{9EA_{rc}\varepsilon_u(EA_{rc} - 2P_{yt}) + EA_{rc}bcf_c(42\varepsilon_u - 18\varepsilon_0) + 9P_{yt}^2} - EA_{rc}\varepsilon_u\right)}{bf_c(3\varepsilon_0 - 7\varepsilon_u)} \tag{3}$$

Substituting Eq.2 and 3 into Eq.1

$$K_{MR} = \frac{2EI}{\tan\alpha\left(\frac{2EI\varepsilon_u}{\tan\alpha} + LP_{yt}d - Lbd_{NA}^2 f_c - EA_{rc}Lc\varepsilon_u + \frac{EA_{rc}Lc^2\varepsilon_u}{d_{NA}} + \frac{13Lbd_{NA}^2 f_c\varepsilon_0}{12\varepsilon_u} - \frac{Lbd_{NA}^2 f_c\varepsilon_0^2}{3\varepsilon_u^2}\right)} \tag{4}$$

Equation 4 can be considered as a mechanics based replacement of current empirically derived moment redistribution factors in national standards that are frequently based on the neutral axis depth factor d_{NA}/d. It can be seen that the currently used parameter has little relationship to the mechanics based parameter given by Eq. 1 and 4. This explains the huge discrepancies between current approaches as well as highlights the difficulty of trying to find an empirical solution to this complex problem.

CONCLUSIONS

Although important in design, moment redistribution has proved to be difficult to quantify due to the complexity of quantifying hinge rotations. In this paper it has been shown how, closed form, mechanics based equations to quantify the moment rotation characteristics of reinforced concrete hinges as well as to quantify the moment redistribution factor can be determined. These equations are an improvement on the empirical hinge formulations in that they simulate what is seen in practice through the simulation of partial interaction mechanisms.

ACKNOWLEDGMENTS

The financial support of the Australian research Council ARC Discovery Project DP140103525 “A new generic approach for assessing blast effects on reinforced concrete members’ is gratefully acknowledged.

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