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DEFORMATION PREDICTION OF SHEAR CRITICAL RC BEAMS BASED ON A MULTI-ANGLE TRUSS MODEL

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ABSTRACT

Deformations of reinforced concrete (RC) beams with a low span-depth ratio can be attributed equally to flexural and shear components. The former can be calculated using well-established theories and the latter can be analyzed by truss analogy while the prediction accuracy largely depends on the assumed diagonal crack angle. In existing truss models, the moment-induced flexural drift effect is usually ignored and consequently the angles of diagonal cracks are underestimated. To correct this deficiency, a multi angle truss model is proposed in this paper to predict the variation of diagonal crack angles along the shear span by considering the flexural drift effect. Nine RC beams with different shear span-to-effective depth ratios were tested to verify the theoretical models. Results show that the proposed multi angle truss model can provide a sound prediction of the deformation of shear critical RC beams.

KEYWORDS

Shear deformation, RC beams, truss model, diagonal crack angle.

INTRODUCTION

Deformation of reinforced concrete (RC) beams is an important measure of their serviceability performance, which is specifically required in current performance-based design codes. Generally, deformation of RC beams consists of flexural deformation and shear deformation. However, compared with the maturity of theoretical models for flexural deformation prediction, such as ACI Building Code (ACI 318-2008), there are few models available for predicting the shear deformation. It is well known that the shear deformation is no longer negligible particularly after the initiation of shear cracks in RC beams with a low span-depth ratio (e.g., Timoshenko et al., 1984)

The truss analogy, firstly introduced by Ritter and Morsch nearly 100 years ago (Ritter, 1899; Mörsch, 1920; and Mörsch, 1922) is considered as a convenient tool for estimating the shear deformation of RC beams. However, due to the neglection of the flexural drift effect along the shear span, most of these formulas, if not all, underestimated the diagonal crack angle, resulting in an overestimation of the shear stiffness. Therefore, in this paper a multi angle truss model (MATM) is proposed to predict the shear deformation of RC beams with consideration of the flexural drift effect along the shear span. In this model, several diagonal cracks with different angles and lengths will be incorporated. Experimental tests will be also conducted on RC beams with lower beam span-effective depth ratios to validate the proposed theoretical model.

TRUSS MODELS ANALOGY

The constant angle truss model (CATM) assumes that the compression struts are parallel to the direction of cracking and the average shear stresses are resisted by the diagonal compressive stresses of concrete. The shear stiffness of the cracked concrete member due to the CATM mechanism is (Mander et al., 1999)

$$K_V = \frac{n\rho_v \cot^2\alpha}{1+n\rho_v \csc^4\alpha} E_c A_v \tag{1}$$

and then the shear deformation is expressed as

$$\delta^1_{s,p} = \frac{V}{K_v} a \tag{2}$$

where V is shear force; A_v=bjd is the effective shear area, in which b is web width of beam and jd is the distance between compressive reinforcement and tensile reinforcement; ρ_v is the shear reinforcement ratio, taken as A_{sv}/bs; s is spacing of stirrups; n= E_s/E_c is the modulus ratio of shear reinforcement to concrete, in which E_s is the modulus of elasticity of shear reinforcement and E_c is the modulus of elasticity of concrete; α is the diagonal crack angle and a is the length of shear span.

Kupfer (1964) pointed out that the diagonal crack angle occurs at the orientation that requires minimum energy dissipation. Based on this assumption, Kim and Mander (1999) derived the following equation to determine the diagonal crack angle:

$$\cot\alpha = \left(\left(\frac{\rho_v}{\rho_t}\right)\left(\frac{A_v}{A_g}\right)\Big/ 0.61\Lambda\right)^{-0.25} \tag{3}$$

where A_g is gross section area; ρ_t is volumetric ratio of longitudinal steel to concrete; and Λ is a factor considering the beam end fixed condition, $\Lambda = 1$ for a fixed pinned condition.

MULTI ANGLE TRUSS MODEL

It has been recognised that the diagonal crack angle α is dependent on compatibility requirements and have to satisfy the equilibrium requirements as well (Vecchio et al., 1988). Therefore, the longitudinal strain distribution of concrete beams (i.e., flexural drift angle) together with equilibrium conditions should be taken into account in determining the diagonal crack angle and shear deformation. Some approaches based on fiber analysis or finite element modelling have been proposed to address the above issues. For instance, Vecchio et al. (1988) suggested that this problem be solved by considering the beam to be composed of a series of concrete layers and longitudinal steel elements. However, these models are usually complicated and implicit compared with truss models and thus they are not convenient for use in practice. Therefore, the below section aims to present an improved truss model。

Figure 1 is a free body diagram of a diagonal crack in a concrete beam subjected to concentrated load V with a projection length of L in the longitudinal direction. Moments at the sections A and B are M_A and M_B respectively. Introducing $\lambda_x = L_x/jd$ $(M_A = V\lambda_x jd)$, the external work (EWD_F) done by the shear force and moments due to flexural deformation is

$$EWD_F = \frac{1}{2}\frac{V^2}{nE_cA_n}\left(\lambda_x^2\cot\alpha + \lambda_x\cot\alpha + \frac{\cot^3 a}{3}\right)jd \tag{4}$$

where $A_n = A_s^c A_s^t/(A_s^c + A_s^t)$; $A_s^c = (bd_c)/\mathrm{n} + A_s^c$; d_c is depth of the compressive zone of the section; A_s^c is the area of compressive steel reinforcement; and A_s^t is area of tensile steel reinforcement.

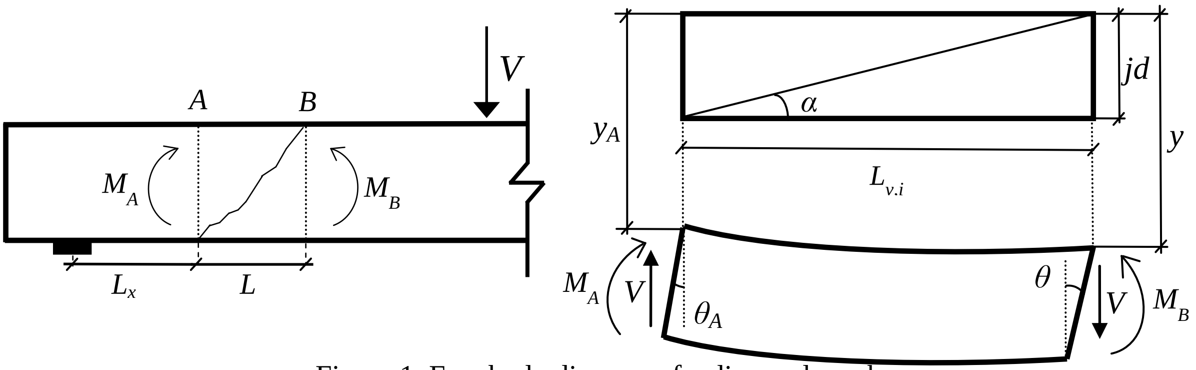

Figure 1. Free body diagram of a diagonal crack

Moreover, the external work done (EWD_v) due to shear deformation is

$$EWD_v = \frac{1}{2}\frac{V^2}{K_v}L \tag{5}$$

Therefore, the total external work done (EWD) is

$$\begin{aligned} EWD = {} & \frac{1}{2}\frac{V^2}{nE_cA_n}\left(\lambda_x^2\cot\alpha + \lambda_x\cot^2\alpha + \frac{\cot^3 a}{3}\right)jd \\ & + \frac{1}{2}\frac{V^2}{nE_cA_v}\left(2n\cot\alpha + n\cot^3\alpha + \frac{1+n\rho_v}{\rho_v}\tan\alpha\right)jd \end{aligned} \tag{6}$$

Differentiating Eq. 6 with respect to α, let $d(EWD)/d\alpha = 0$, the potential diagonal crack angle is governed by

$$\left(\frac{3n}{A_v}+\frac{1}{A_n}\right)\cot^4\alpha + \frac{2\lambda_x}{A_n}\cot^3\alpha + \left(\frac{\lambda_x^2}{A_n}+\frac{2n}{A_v}\right)\cot^2\alpha - \frac{1+n\rho_v}{A_v\rho_v} = 0 \tag{7}$$

Figure 2 shows an exemplary solution of Eq. 7 with assuming A_v=6.57(10^{4}mm^2), A_v=9.79(10^{2}mm^2), n=5.68, $\rho_v = 0.32\%$. It can be seen that the angle of diagonal crack is no longer a constant but increases along the shear span. In addition, according to the Modified Compression Field Theory (MCFT) (Collins et. al., 1991), the shear force can be expressed as

$$V = f_1 bjd\cot\alpha + \frac{A_v f_v}{s} jd\cot\alpha \tag{8}$$

where f_1 is principal tensile stress in concrete and f_v is the stress in the shear reinforcement. Since $f_1 \leq f_t$ and $f_v \leq f_{yv}$, cotα should have a lower bound value:

$$\cot\alpha \geq V \Big/ \left(f_t bjd + \frac{A_v f_{yv}}{s} jd\right) = \cot\alpha_l \tag{9}$$

where f_t is concrete tensile strength, taken as 0.333$(f_c')^{0.5}$ in which f_c' is compressive strength of concrete cylinder and f_{yv} is yielding stress of shear reinforcement.

Generally, some diagonal cracks can extend up to the concentrated loading point when RC beams fail in shear. Therefore, if assuming that the loading point is the end of the first diagonal crack and drawing a line geometrical *cotα* (*y=x/jd*), where *x* is the distance from loading point, an intersection point can be obtained from the solution of Eq. 7 and Eq. 9. At this point, the *cotα* on the curve line is equal to geometrical *cotα*, which means that a diagonal crack occurs from this point (see Figure 2). When the first diagonal crack is found out, adopting the same strategy but starting from the first intersection point, the next diagonal crack will be obtained subsequently. Repeating the same procedures until the line of geometrical *cotα* cannot intersect with the curve of potential *cotα*. Finally, all the actual diagonal crack angles (i.e. *cotα*) and corresponding lengths can be obtained, as shown in Fig.2.

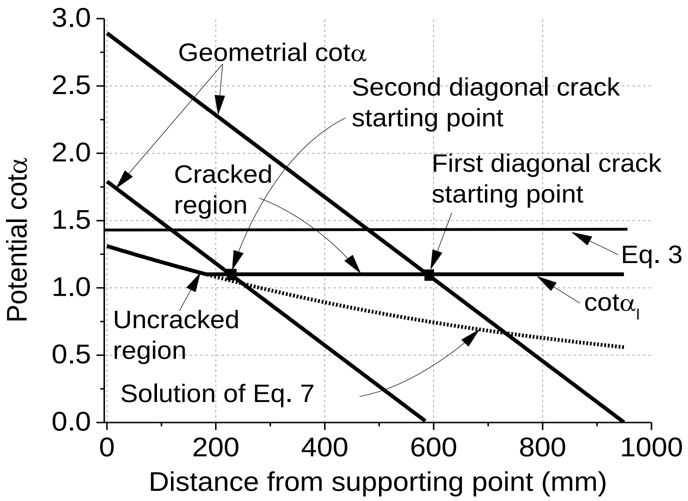

Figure 2. Multi-angle truss model

It should be noted that, although there is an un-cracked region, the diagonal cracks will eventually extend to supporting point at the ultimate shear load. Therefore, this region should be also considered. For the sake of simplification, it is assumed that the shear stiffness of this region is also determined by Eq. 1 but the corresponding *cotα* is the average *cotα* of this region. Finally, according to the result of MATM, the shear deformation $\delta_{s,p}^2$at the ultimate shear load is

$$\delta_{s,p}^2 = \sum_1^n \frac{V}{K_v^i} L_v^i \tag{10}$$

where K_v^i is the shear stiffness of the i^{th} diagonal crack; L_v^i is the length of the i^{th} diagonal crack; and n is the number of diagonal cracks.

EXPERIMENTAL VALIDATION

Descriptions of Tests

To verify the effectivenss of the multi angle truss model, a test program consisting of nine RC beams was conducted to investigate the deformation of shear critical beams (Wang 2012). The variables in this experiment included the shear span-to-effective depth ratio, the shear reinforcement ratio and the concrete strength. All the test beams had the section of 200 mm × 400 mm and the concrete cover depth was 25 mm. The details of the beams are listed in the Table 1. The yielding stresses of the longitiudal and shear reinforcment were 584 MPa and 537 MPa, respectively. These RC beams were symmetrically loaded with two concentrated point loads and the corresponding shear span for each beam is listed in Table 1. Instrumentations were provided to measure the loads and the beam deflections. In this test, five Linear Variable Differential Transformers (LVDTs) were installed to measure the deflections below the mid-span, two load points, and two supports. Before the formal tests, a preload of 30 kN was applied on the beams to ensure that all the instruments functioned well.

Table 2 lists the shear deformaiton, $\delta_{s,e}$, of all the tested beams, which was calculated by subtracting the total deflection with the flexural deflection predicted by ACI318-08 (2008) at the ultimate state and the corresponding ultimate shear force V_u are both given in Table 2. More details about the experiments can be found in Wang (2012).

Table 1. Details of reinforced concrete beams

Beam No.	Span (mm)	λ	f_{cu} (MPa)	E_c (10^4MPa)	Tensile reinforcement	Compressive reinforcement	Strirrup ratio (%)	Shear reinforcement
B1	640	1.75	24.2	3.1	2D25+1D16	2D18	0.39	D8@130
B2	820	2.25	24.2	3.1	2D25+1D16	2D18	0.25	D8@200
B3	950	2.60	25.2	3.1	3D25	2D25	0.39	D8@130
B4	640	1.75	42.5	3.6	2D25	2D18	0.25	D8@200
B5	640	1.75	25.2	3.1	2D25	2D18	0.32	D8@160
B6	820	2.25	50.0	3.6	3D25	2D18	0.39	D8@130
B7	820	2.25	25.2	3.1	2D25+1D16	2D18	0.32	D8@160
B8	950	2.60	50.0	3.6	3D25	2D18	0.32	D8@160
B9	950	2.60	24.2	3.1	2D25+1D16	2D18	0.25	D8@200

Note: f_{cu} is the compressive strength of concrete 150mm cube; λ is the shear span-to-effective depth ratio.

Table 2. Experimental result of test beams

Beam No.	B1	B2	B3	B4	B5	B6	B7	B8	B9
V_u (kN)	219	185	262	215	242	300	217	275	203
$\delta_{s,e}$ (mm)	3.382	3.835	5.045	3.174	4.084	3.731	3.928	4.734	5.105

Table 3. Comparison between predicted and experimental shear deformation at the ultimate state (mm)

	B1	B2	B3	B4	B5	B6	B7	B8	B9	Avg.	SD. (%)
cotα	1.296	1.443	1.373	1.378	1.304	1.370	1.365	1.443	1.443	——	——
$\delta_{s,e}$	3.382	3.835	5.045	3.174	4.084	3.731	3.928	4.734	5.105	——	——
$\delta_{s,p}^1$	1.882	2.474	3.090	2.387	2.466	2.958	2.623	3.460	3.142	——	——
$\delta_{s,p}^2$	2.912	3.622	4.181	3.506	2.936	3.874	3.591	5.129	4.639	--	--
$\delta_{s,e}/\delta_{s,p}^1$	1.797	1.550	1.632	1.330	1.656	1.261	1.497	1.368	1.625	1.524	16.547
$\delta_{s,e}/\delta_{s,p}^2$	1.161	1.059	1.207	0.905	1.391	0.963	1.094	0.923	1.101	1.089	14.472

Experimental Results vs. Model Predictions

Using Eq. 2 and Eq. 10, the shear deformation of each test beam could be obtained using both the constant angle truss model (CATM) and the multiple angle truss model (MATM). Table 3 presents the comparison between the experimental results and the two models predictions. It can be seen that the CATM leads to an underestimation of the shear deformation. The average ratio of the experimental results to the predicted values is 1.524. This is mainly because that the diagonal crack angles were underestimated (refer to Fig.2). The flexural drift effect in the beams caused by the moment change along the shear span tends to change the crack pattern from "diagonal cracks" to "flexural cracks" along shear span. Therefore, such an effect should be taken into account in determining the diagonal crack angle. Obviously, when the effects of flexural drift are taken into account, MATM provides a much better prediction of the shear deformation of RC beams with low span-depth ratios. It can be seen from Table 3 that although the standard deviations of two predicted models are still similar (i.e. CATM and MATM), the average ratio of the test results to predicted values is significantly reduced from 1.524 to 1.089, which shows that effects of flexural drift is very significant.

CONCLUSIONS

The shear deformation of loaded RC beams with lower span-to-effective depth ratios is significant and it is necessary to develop reasonable theoretical model to predict such deformation. It was found that conventional constant angel truss models underestimate the shear deformation since they neglect the flexural drift effect caused by the moment change along the shear span and thus underestimate the diagonal crack angle. A new multi angle truss model has been proposed in the paper to take into account the effect of flexural drift. The proposed model has been validated through comparisons with the experimental shear deformations obtained from the tests of nine RC beams with different shear span-to-effective depth ratios, different concrete strength and different shear reinforcement ratios.

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