Southern Cross University
ePublications@SCU

23rd Australasian Conference on the Mechanics of Structures and Materials

2014

# Development of the compressive membrane action in partially-restrained reinforced concrete sub-assemblages

N Vessali
*University of Technology Sydney*

H Valipour
*University of New South Wales*

B Samali
*University of Western Sydney*

S J. Foster
*University of New South Wales*

## Publication details

Vessali, N, Valipour, H, Samali, B & Foster, SJ 2014, 'Development of the compressive membrane action in partially-restrained reinforced concrete sub-assemblages', in ST Smith (ed.), *23rd Australasian Conference on the Mechanics of Structures and Materials (ACMSM23)*, vol. I, Byron Bay, NSW, 9-12 December, Southern Cross University, Lismore, NSW, pp. 357-362. ISBN: 9780994152008.

ePublications@SCU is an electronic repository administered by Southern Cross University Library. Its goal is to capture and preserve the intellectual output of Southern Cross University authors and researchers, and to increase visibility and impact through open access to researchers around the world. For further information please contact epubs@scu.edu.au.

# DEVELOPMENT OF THE COMPRESSIVE MEMBRANE ACTION IN PARTIALLY-RESTRAINED REINFORCED CONCRETE SUB-ASSEMBLAGES

**N. Vessali***
School of Civil and Environmental Engineering, University of Technology Sydney
PO Box 123, Broadway, NSW, 2007, Australia. nima.vessali@student.uts.edu.au
(Corresponding Author)

**H. Valipour**
School of Civil and Environmental Engineering, the University of New South Wales
UNSW, Sydney, NSW, 2052, Australia. h.valipour@unsw.edu.au

**B. Samali**
Institute for Infrastructure Engineering, University of Western Sydney
Penrith, NSW, 2751, Australia. b.samali@uws.edu.au

**S.J. Foster**
School of Civil and Environmental Engineering, the University of New South Wales
UNSW, Sydney, NSW, 2052, Australia. s.foster@unsw.edu.au

## ABSTRACT

Development of compressive membrane (arching) action in longitudinally-restrained reinforced concrete (RC) beams subjected to medium to large displacements can enhance the stiffness, ultimate load capacity and the robustness of the RC frames under unexpected loading scenarios. The enhancement provided by the arching action is not considered in the current design practice and accordingly, the main objective of this paper is to evaluate the development of compressive membrane action in partially-restrained RC beam sub-assemblages. The results of experiments conducted on RC sub-assemblages with different concrete compressive strength ($f'_c$) and reinforcement ratio ($\rho$) are briefly discussed and the tested sub-assemblages are analysed using two different types of nonlinear FE models, i.e. 2D continuum-based and 1D frame FE models. The results obtained from the numerical models as well as the experimental data are employed to quantify the enhancing effects of the compressive membrane action with respect to concrete compressive strength and reinforcing bar proportion.

## KEYWORDS

Arching action, compressive strength, reinforcement ratio, nonlinear FE analysis, partially-restrained RC sub-assemblage.

## INTRODUCTION

Reinforced concrete (RC) flexural members can exhibit higher load-bearing capacity than what is predicted from simple plastic analysis (Johansen, 1943), owing to development of compressive membrane (arching) action. In spite of the extensive experimental studies conducted on RC slabs (Ockleston, 1955, Park, 1964, Kennan, 1969, Brotchie and Holley, 1971, Taylor et al., 2001), development of the arching action in RC frames has not been investigated sufficiently. The available

This work is licensed under the Creative Commons Attribution 4.0 International License. To view a copy of this license, visit http://creativecommons.org/licenses/by/4.0/

experimental data on RC framed structures confirm that the compressive strength of concrete and reinforcement ratio have beneficial effects on the strength enhancement provided by arching action (Su et al., 2009, Yu and Tan, 2012).

Apart from experimental studies, some attempts have been made to develop simplified analytical models for capturing the arching action in RC members. However, such simplified methods suffer from shortcomings such as inability to take the support stiffness into account, inability to predict the complete load-deflection diagram and also adopting unrealistic assumptions for concrete behaviour (e.g. rigid-plastic). Accordingly, some researchers have resorted to finite element (FE) models to capture the membrane behaviour in RC members (Izzuddin and Elghazouli, 2004, Valipour and Foster, 2010a). However, due to the numerical complexities associated with geometrical nonlinearities as well as cracking and crushing of the concrete, only few FE models have been successful in capturing the membrane action in RC frames. Also, the spurious mesh sensitivity associated with concrete softening under compression is still a big challenge in the context of the continuum-based models.

The results of the experiments on six partially-restrained RC sub-assemblages which were categorised on the basis of compressive strength of concrete and the reinforcement ratio have been presented in this paper. Two different nonlinear FE models, i.e. 2D continuum-based and 1D discrete frame have been used to simulate the behaviour of RC sub-assemblages subject to column loss and it is shown that the developed FE models can adequately capture the peak load capacity and load-deflection response of members developing arching action. The numerical and experimental results were employed to determine the relationship between compressive strength of concrete and arching action.

## EXPERIMENTAL STUDY

Six RC beam sub-assemblages representing the behaviour of a frame following loss of a middle column were tested under a monotonically increasing downward displacement applied at the centre stub. The specimens were categorised with respect to concrete compressive strength and reinforcement ratio (Figure 1 and Table 1). The specimens were partially restrained against translation and rotation at both ends by bollards anchored to the reaction floor (Figure 2). The translational as well as rotational movement of the supports were measured by using LVDTs and inclinometers, respectively.

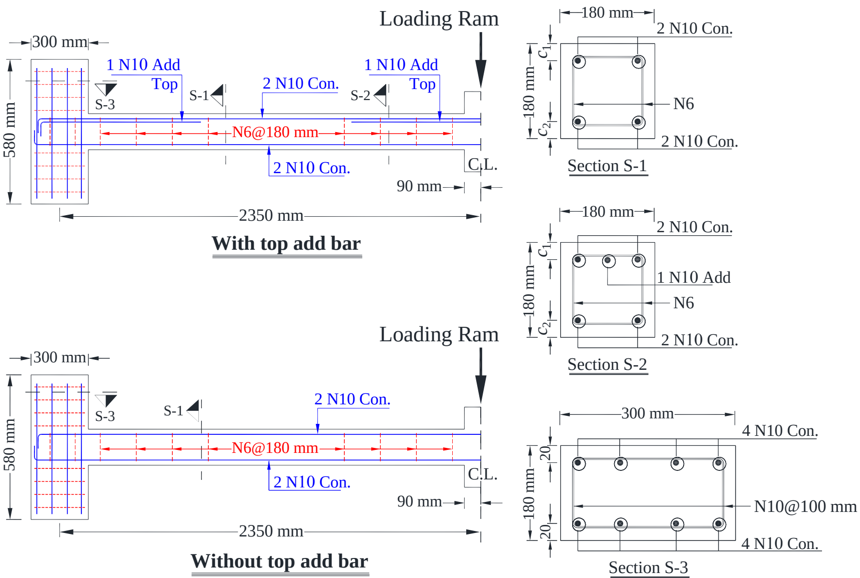


Figure 1. Outline of geometry, cross-section and reinforcing details for RC beam sub-assemblages.

Table 1. Concrete compressive strength, reinforcement details and concrete cover for sub-assemblages

| Specimen No. | Concrete Compressive Strength (MPa) | | Existence of Top add bar | Concrete Cover (mm) | |
|---|---|---|---|---|---|
| | 28th Day | Testing Day | | $c_1$ | $c_2$ |
| 1 | 59 | 67 | Yes | 30 | 37 |
| 2 | | | No | 37 | 24 |
| 3 | 42 | 48 | Yes | 27 | 33 |
| 4 | | | No | 30 | 36 |
| 5 | 13 | 18 | Yes | 37 | 21 |
| 6 | | | No | 33 | 30 |

The normal ductility 10 mm ribbed bars with nominal strength of 500 MPa were used for longitudinal reinforcement and 6 mm hoops fabricated from round bars and of nominal strength of 250 MPa were used for transverse reinforcement. The stress-strain curve of the 10 mm ribbed bars was obtained from direct tension test (Figure 3a). Three different concrete grades (20 MPa, 40 MPa and 60 MPa) were used in specimens and the compressive strength of each specimen was determined on the testing date (see Table 1). Furthermore, the stress-strain curve of the three different concrete grades was obtained from uniaxial compression test on the concrete cylinders (Figure 3b).

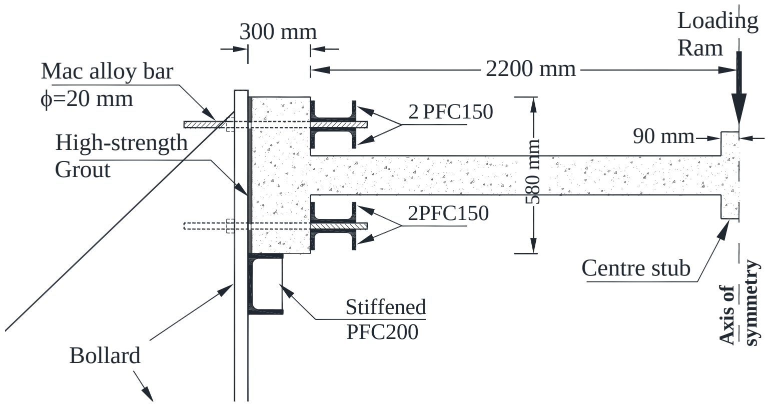



Figure 2. Experimental set up for the reinforced concrete sub-assemblages.

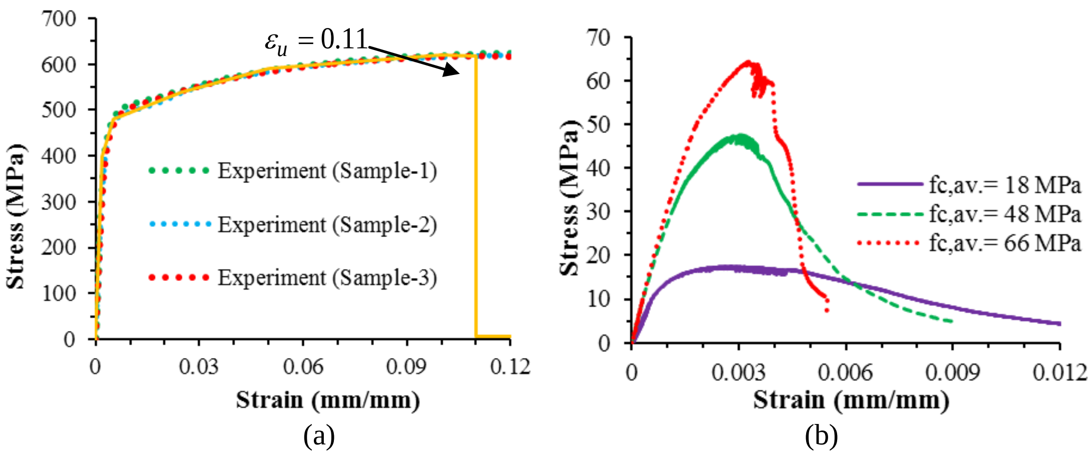
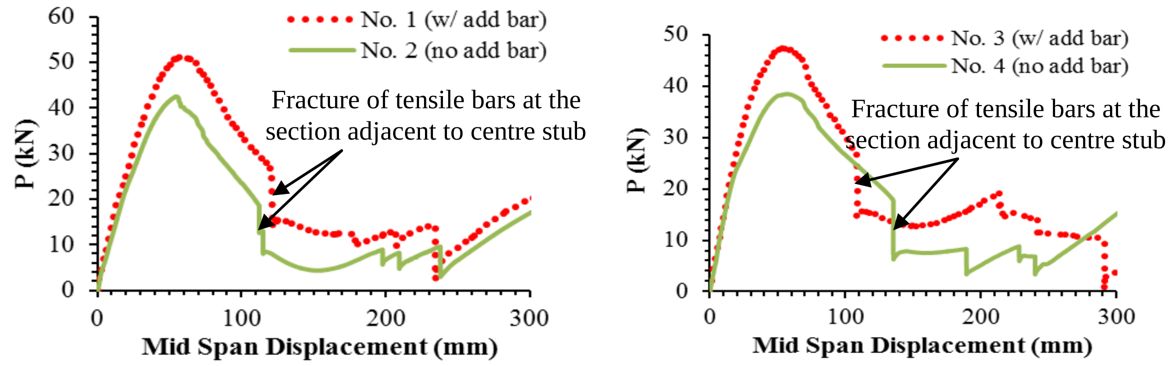



Figure 3. Uniaxial stress-strain diagram for (a) N10 longitudinal steel rebar under tension (b) concrete (300 by 150 mm diameter cylinder) under compression.

**Test Results**

In the specimens with the top add bars (i.e. No. 1, 3 & 5), it was observed that first major cracks developed at sections adjacent to the centre stub. For the specimens without the top add bars, cracks at sections adjacent to centre stub and end block took place around the same load level. In all sub-assemblages, the first rupture of tensile bars was observed in sections adjacent to centre stub. The applied load versus vertical displacement of the centre stub for sub-assemblages No.1 to 4 are shown in Figure 4; the sharp drops in these load-displacement diagrams are indicative of bar rupture.




(a) (b)

Figure 4. Load-displacement for sub-assemblages with (a) 60 MPa and (b) 40 MPa concrete grade.

## FINITE ELEMENT ANALYSIS

### 1D Discrete FE Model

The strain-displacement compatibility equation for the generic beam is derived from Navier- Bernoulli hypothesis and principle of virtual work for a simply supported configuration (Valipour et al., 2013),

$$\bar{\mathbf{q}}' = \int_0^l \bar{\mathbf{b}}^{\mathrm{T}}[x, w(x)]\,\mathbf{d}(x)\,dx \tag{1}$$

where $\bar{\mathbf{q}}'$ and $\mathbf{d}(x)$ are the generalised nodal displacement and section strain vectors, respectively and $\bar{\mathbf{b}}^{\mathrm{T}}[x, w(x)]$ is the interpolating function matrix. The equilibrium equation for the generic model is

$$\bar{\mathbf{D}}(x) = \mathbf{b}[x, w(x), \theta(x)]\,\bar{\mathbf{Q}} + \bar{\mathbf{D}}^*(x) \tag{2}$$

where $\mathbf{Q}$ denotes the nodal force vector, $\bar{\mathbf{D}}(x)$ and $\bar{\mathbf{D}}^*(x)$are the section force vector and the section force vector due to the element load respectively and $\mathbf{b}[x, w(x), \theta(x)]$ is a force interpolation matrix. Combining equations (1) and (2) and decomposing the strain and displacement vectors to their elastic and plastic components, yields (Valipour et al., 2013)

$$\left\{\mathbf{T}^{\mathrm{T}}\mathbf{K}\,\mathbf{T}^*\right\}\mathbf{q} = \mathbf{Q} + \mathbf{T}^{\mathrm{T}}\mathbf{K}\left(\bar{\mathbf{q}}_p + \bar{\mathbf{q}}_{pr}\right) + \left(\mathbf{Q}^* + \mathbf{T}^{\mathrm{T}}\mathbf{K}\,\bar{\mathbf{q}}^*\right) \tag{3}$$

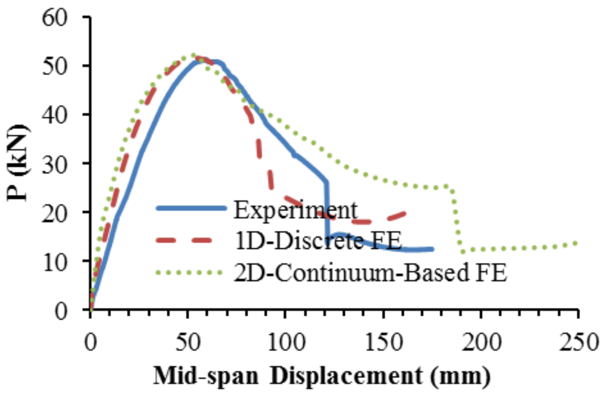
where $\mathbf{K}$ is the stiffness matrix for the simply supported configuration including the nodal springs, $\mathbf{Q}$ and $\mathbf{q}$ respectively, denote the nodal force and displacement vectors in the system with rigid body mode. $\bar{\mathbf{q}}_p$, $\bar{\mathbf{q}}_{pr}$ and $\bar{\mathbf{q}}^*$ are respectively the nodal generalised plastic deformation vector excluding nodal springs, the generalised plastic deformation vector for nodal springs and the nodal generalised deformation vector due to member load. Also, $\mathbf{T}$ and $\mathbf{T}^*$are the transformation matrices which relate the force and the displacement vectors of the systems with and without rigid body motion, respectively (Valipour and Foster, 2010b). The CEB-FIP model was employed for hardening of concrete in compression and for softening of concrete under compression a linear softening model was used. For the tensile concrete, a linear elastic brittle failure model was adopted. Furthermore, in the proposed compound element, nodal springs with damage multipliers are employed to capture the fracture of longitudinal bars.

### 2D Continuum-Based FE Model

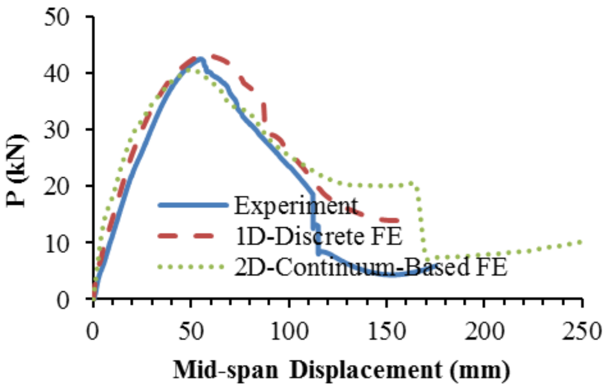
The beam sub-assemblages and the bollards were also modelled and analysed using a 2D plane stress FE model in ATENA software (Cervenka et al., 2002). In the framework of total secant damage a hypoelastic material model called SBETA was employed to model the concrete behaviour. The model is capable of capturing concrete nonlinearities (i.e. cracking and crushing). The size of the elements in the 2D FE mesh was limited to 10 mm and geometrical nonlinearities were taken into account using updated Lagrangian formulation (Cervenka et al., 2005). The constitutive law of the reinforcing bars was modelled by a multi-linear elastic-plastic hardening behaviour with a sharp drop in the stress-strain curve at a uniform elongation of $\varepsilon_u = 0.11$ to incorporate the bar fracture in the FE model.

## RESULTS AND DISCUSSIONS

The load versus vertical displacement of centre stub captured by both discrete and continuum-based FE models, for specimens Nos. 1 and 2 are shown in Figure 5. The peak load capacity of the beam sub-assemblages observed in the tests and that predicted by FE models are given in Table 2. It is seen that the load-deflection response and the peak load capacity predicted by FE models correlates well with the test data; however, the peak loads predicted by 1D discrete FE model are closer to those of the experiments. Furthermore, the peak loads exhibited by both models are higher than the one calculated from the simple plastic analysis (see Table 2).




(a)




(b)

Figure 5. Load-displacement of sub-assemblages (a) No.1 and (b) No. 2 predicted by FE models.

Table 2. Peak load capacity of the specimens obtained from test data and finite element models.

| Specimen No. | Peak Load (kN) | | | |
|---|---|---|---|---|
| | Test | 1D Discrete FE | 2D Continuum-Based FE | Plastic Hinge |
| 1 | 51.3 | 51.7 | 52.0 | 31.52 |
| 2 | 42.5 | 43.3 | 40.7 | 25.46 |
| 3 | 47.4 | 47.5 | 47.5 | 30.98 |
| 4 | 38.5 | 38.7 | 38.0 | 25.13 |
| 5 | 33.3 | 32.5 | 32.5 | 27.82 |
| 6 | 28.3 | 28.6 | 27.8 | 23.19 |

## CONCLUSIONS

A set of six 2/5th scale RC beam sub-assemblages with partially-fixed boundary conditions were constructed and tested under a monotonically increasing push-down displacement applied at the centre stub. The concrete compressive strength and longitudinal reinforcing proportion were the main variables within different specimens. In addition to the experimental program, two nonlinear FE models with exact boundary conditions as per test setup were developed and analysed. The end-support stiffness of beam sub-assemblages was incorporated into the FE models by using translational as well as rotational springs at supports in 1D discrete FE model and by modelling the bollards, PFCs and anchorage bars in 2D continuum-based model. The following conclusions are drawn from the results of the experimental and FE results in this study;

- It is known that the compressive strength of concrete typically has only a minor influence on peak load capacity of slender (flexure-dominant) beams such as the ones tested in this study. Accordingly, the fairly significant influence of concrete compressive strength on the capacity of the tested beams can be attributed to development of arching action.

- For the RC beam sub-assemblages tested in this study, a nearly linear relationship between the load capacity and compressive strength of the concrete was observed. This observation is consistent with that seen in the experimental studies on slab strips (Taylor and Mullin, 2006) as well as results of numerical parametric studies on beam sub-assemblages (Valipour et al., 2013).
- The longitudinal reinforcing proportion had significant influence on the peak load capacity of the tested RC sub-assemblages. However, the reinforcing proportion had a negligible influence on the enhancing effect of arching action.

## ACKNOWLEDGMENTS

This project was funded by ARC Discovery Grant DP120103328; the support of the ARC is acknowledged with thanks.

## REFERENCES

Brotchie, J. F. & Holley, M. J., (1971). "Membrane Action in Slabs, Cracking, Deflection, and Ultimate Load of Concrete Slab Systems". *ACI Special Publication SP-30*,, pp. 345-377.

Cervenka, V., Cervenka, J. & Pukl, R., (2002). "ATENA - A tool for engineering analysis of fracture in concrete". *Sadhana - Academy Proceedings in Engineering Sciences,* Vol. 27**,** No. 4, pp. 485-492.

Cervenka, V., Jendele, L. & Cervenka, J., (2005). "ATENA Program Documentation, Part 1: Theory". Prague.

Izzuddin, B. A. & Elghazouli, A. Y., (2004). "Failure of lightly reinforced concrete members under fire. I: Analytical modeling". *Journal of Structural Engineering,* Vol. 130**,** No. 1, pp. 3-17.

Johansen, K. W. (1943) *Yield-line formulae for slabs,* London, Cement and Concrete Association.

Kennan, W. A., (1969). "Strength and behavior of restrained reinforced concrete slabs under static and dynamic loadings". *Report No.* R621, U.S. Naval Civil Engineering Laboratory, Port Hueneme, CA.

Ockleston, A. J., (1955). "Load tests on a three-storey reinforced concrete building in Johannesburg". *Structural Engineer,* Vol. 33**,** No. 10, pp. 304-322.

Park, R., (1964). "Ultimate strength of rectangular concrete slabs under short term uniform loading with edges restrained against lateral movement.". *Proceedings of Institution of Civil Engineers,* Vol. 28**,** No. June, pp. 125-150.

Su, Y., Tian, Y. & Song, X., (2009). "Progressive collapse resistance of axially-restrained frame beams". *ACI STRUCTURAL JOURNAL,* Vol. 106**,** No. 5, p. 600.

Taylor, S. E. & Mullin, B., (2006). "Arching action in FRP reinforced concrete slabs". *Construction and Building Materials,* Vol. 20**,** No. 1-2, pp. 71-80.

Taylor, S. E., Rankin, G. I. B. & Cleland, D. J., (2001). "Arching action in high-strength concrete slabs". *Proceedings of the Institution of Civil Engineers: Structures and Buildings,* Vol. 146**,** No. 4, pp. 353-362.

Valipour, H., Farhangvesali, N. & Foster, S., (2013). "A generic model for investigation of arching action in reinforced concrete members". *Construction and Building Materials,* Vol. 38**,** No. 0, pp. 742-750.

Valipour, H. R. & Foster, S., (2010a). "Progressive collapse analysis of planar reinforced concrete frames using 1D distributed nonlinearity element". *Australian Journal of Structural Engineering,* Vol. 10**,** No. 3, pp. 263-275.

Valipour, H. R. & Foster, S. J., (2010b). "Finite element modelling of reinforced concrete structures including catenary action". *Computers and Structures,* Vol. 88**,** No. 9, pp. 529-538.

Yu, J. & Tan, K. H., (2012). "Structural Behavior of RC Beam-Column Sub-assemblages under a Middle Column Removal Scenario". *Journal of Structural Engineering,* Vol. 139**,** No. 2, pp. 233-250.