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Robust relationships for simple plantation growth models based on sparse data

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Abstract

Three equations predicting height $H=\beta_1(t-0.5)^{0.5}$, diameter $D = \beta_2(H-1.3)/lnN$, and mortality $dN/N= -2(G/ G_{max})^3 dD/D$ from plantation age (t), stocking (N) and basal area (G) can be calibrated with few data (even a single observation) for plantations in which re-measured data and growth models are unavailable. Despite having only three parameters to be estimated, these equations extrapolate reliably and allow objective forecasts of future plantation growth performance that may serve as useful first approximations until more precise growth models can be developed.

Keywords: forest growth model, plantation, even-aged

Introduction

Well-known forest growth models are often based on very large (e.g., Buckman et al., 2006; Vanclay, 1994a) or sophisticated databases (e.g., Landsberg et al., 2003; Battaglia et al., 2004), but there remains strong demand for forest growth forecasts in situations where efforts to calibrate and initialize models are hampered by a lack of data. This paper examines some robust principles that may underpin simple models based on minimal data for forest plantation forest forecasting.

Yield tables may be the oldest and most robust approach to yield prediction in forest plantations, but are severely constrained by the need to follow a standard management regime. In many situations where decision support is needed, plantation management regimes may not be standardised, and may be far from optimal. Furthermore, the demand for a growth model may be to explore harvesting and management options, not to lock in a prescribed production regime. Thus this paper examines dynamic growth models, with minimal attention to yield tables and other static approaches to plantation yield forecasting.

There are several ways to construct a robust dynamic model. One robust way is to model the growth in height and diameter of the average stem, and to estimate the stem size distribution and likely mortality at the stand level. Many other approaches are possible (including models based on the total volume production, on the stem size distribution, or based on physiology), but these tend to require more data to calibrate, and may be more complicated to understand and to verify.

Robust approaches to forecast timber production with sparse data have previously been examined (e.g., Turin, 1913; Zeide, 1978), but it is timely to re-examine simple robust methods for predicting tree growth in smallholder plantings, revegetation programs, and other data-poor situations where objective 'rule-of-thumb' forecasts may be helpful.

Site Index and Height Growth of Forest Stands

Site index, expressed as the expected height of a sample of trees at a given age, has long been recognised and used as a practical measure of site productivity (Skovsgaard & Vanclay, 2008). In addition, tree height is an important indicator of log volume and potential products from a plantation enterprise. Thus the simulation of stand height growth tends to be indispensible in a plantation growth model.

Where sufficient data are available, the Schumacher (1939), Chapman-Richards (von Bertalanffy, 1949; Richards, 1959; Chapman, 1961; Wieskittel et al., 2009), and other equations provide good bases for predicting height growth. However, situations where data are limited in number or range, a more-easily calibrated relationship may be required, and the limiting case is a one-parameter model such as $H=\beta t^c$ or $H=\beta ln(t+1)$, where H is top or dominant height, t is age in years, and β is a parameter to be estimated (e.g., Vanclay et al., 2008). Such models are wrong, but may be useful[1] in offering a robust alternative as an interim measure, especially if they are further simplified to a single-parameter relationship by constraining the exponent c as a constant (e.g., as c=0.5) rather than a variable to be estimated. Such uni-variate relationships may provide a good fit to observed data – comparable to that attained with a more complex multi-parameter equation – during the mid-rotation of a plantation, but generally offer an inferior fit at extreme ages (e.g., very young and very old stands). This deficiency may be of little practical consequence, as young stands may not yet reflect site conditions reliably, and once stands approach (or exceed) optimal harvesting age there should exist additional data for re-calibrating better models.

[1] A reference to an often-quoted statement by G.E.P Box (1979) who wrote "All models are wrong, but some are useful" in *Robustness in Statistics*, eds. Launer, R. L. & Wilkinson, G. G. (Academic, New York), p. *202*.

Given the utility of a single-parameter model, the question remains whether $H=\beta t^c$ or $H=\beta ln(t+1)$ is more suited to the plantation forests in question. The lack of an inflection (c.f., Schumacher, 1939; and other s-shaped curves) is less of a problem for many tropical plantation species with rapid early height growth than it may be for temperate species which tend to exhibit a sigmoidal height growth pattern. The two curves under discussion also exhibit marked differences in predicted heights late in a rotation, with the former ($H=\beta t^c$) predicting larger height growth in late-age stands than the latter ($H=\beta ln(t+1)$). In many cases, useful insights may be gained from the corresponding derivatives $dH/dt=\beta^2 H^{-1}/2$ and $dH/dt=\beta(e^{-H/\beta}-1)$ illustrating height growth versus height. In the former case, the graph of tree growth versus size should exhibit an inverse relationship ($\approx H^{-1}$), whereas in the latter case the relationship is curved more strongly ($\approx e^{-H}$). However, these subtleties of mature height growth may emerge only after the economic rotation age of a plantation has been exceeded, when it has little practical relevance. For many tropical tree plantation species, the former offers a reasonable first approximation to observed height growth patterns (Vanclay, 2009a), especially as the variant

$$H=\beta_1(t-0.5)^{0.5} \qquad (1)$$

where H is stand height in metres, t is age in years, and β_1 is a parameter to be estimated. Readers are cautioned that this relationship tends to overestimate in older stands, and in some temperate species (such as *Pinus radiata*) with a more sigmoidal height growth pattern.

Figure 1 illustrates the prediction based on equation 1 for two stands of *Eucalyptus grandis* (Mattay and West, 1994; plots 75 and 88) in Queensland, chosen because of the large number of remeasures and wide span of ages (12 measures spanning 20 years or more). In this case, an extrapolation based on a single observation at age 4 provides a reasonable basis for predicting the next 20 years.

Figure 2 compares the observed and fitted heights, illustrating that equation (1) is a surprisingly good first approximation but the s-shaped trend indicates that a sigmoidal equation would offer a better fit.

Tree Diameter Growth

There are several ways to estimate tree diameters in forest plantations. Two common and contrasting alternatives are to focus on the stand level or the tree level (Vanclay, 1994b). Typically, a stand-level approach will estimate the total stand production and distribute it amongst the component trees to infer individual stem sizes. A tree-level alternative may predict the growth of each individual tree, or of a selection of representative trees, more directly. The former is a robust approach for homogeneous forest stands, but poses challenges in some smallholder situations where plantings are not configured as compact stands.

Vanclay (2009b) documented a simple relationship between diameter (cm), height (m) and stocking (stems/ha) that can provide reliable estimates of tree diameters: $D = \beta_2(H-1.3)/lnN$, where β_2 tends to take a value between 5 and 8. The relationship tends to remain stable over long time periods for any stand (Vanclay, 2009b; Fig. 1). With this relationship, any pair of stand height and stocking observations can be used to estimate the corresponding mean diameter. However, in situations where there may be mortality or thinning, a more robust approach is to estimate the diameter increment in each time period. The derivative leads to

$$dD = \beta_2 dH/lnN - \beta_2((H-1.3)/Ln^2N)(dN/N) \quad (2)$$

where D is stand mean diameter (cm), H is top height (m) and N is stocking (stems/ha). The left part of this equation ($\beta_2 dH/lnN$) reflects the diameter growth in situations where the stocking does not change. The right part of the equation ($-\beta_2((H-1.3)/Ln^2N)(dN/N)$) reflects the potential growth stimulus in the residual stand when trees die or are thinned (since dN/N is negative, this part will increase overall increment).

The beauty of this relationship is that a single observation of mean diameter, stand height and stocking in a plantation can be used to make an objective prognosis of its future development, albeit a first approximation.

Self-thinning

Many smallholder and restoration plantings exhibit some natural mortality, so a model for such situations should simulate self-thinning. One efficient way to model self thinning draws on the approach of Vanclay and Sands (2009) which allows mortality to be predicted as

$$dN/N = -2(G/G_{max})^3 dD/D \quad (3)$$

where N is stand stocking (stems/ha), G is stand basal area (m^2/ha), D is mean diameter (cm dbhob), and G_{max} is the maximum stand basal area for the site-species combination. If no independent estimate is available, G_{max} can be estimated as $G_{max}=G[1-(N/N_0)^3]^{-1/3}$, where G and N are current basal area (m^2/ha) and stocking (stems/ha), and N_0 is the initial stocking at planting. This approach is easily estimated and implemented, and provides realistic estimates of self-thinning (Figure 4).

Testing the equations

These three equations are simple empirical, one-parameter equations intended to provide robust first approximations rather than precise estimates. The equations are inherently constrained to produce reasonable results, and are tolerant of extrapolation. Figures 2, 3 and 4 reveal that although imprecise, the equations have relatively little bias in well-stocked stands typically encountered in production

forestry. Under these circumstances, many of the usual model evaluation procedures (Vanclay and Skovsgaard, 1997) have limited utility, and the most useful test may be examine the implications of the interaction of these equations in complex situations, such as the optimization of thinning prescriptions.

Optimization often reveals weaknesses (Vanclay and Skovsgaard, 1997), and in the present case, was quick to reveal that equation 2 can overestimate for extremes including low (e.g., <100 stems/ha) and high stocking (>10,000 stems/ha) and for heavy thinning (e.g., removing >50% of the stand). Optimization can be done with a simple solver by implementing the three equations in a spreadsheet, but it is necessary to constrain stocking (e.g., $100 \leq N \leq 10000$ stems/ha) and to constrain the right part of equation 2 ($-\beta_2((H-1.3)/Ln^2N)(dN/N)$) so that the thinning response does not become excessive (e.g., constrain the right part so it does not exceed the left part). With these constraints in place, the three equations provided reasonable predictions when optimized for mean annual volume increment and for sawlog production.

Table 1 summarizes some optimization results. These are not definitive and apply to a hypothetical generic species, but illustrate that the three equations are well behaved in simulation studies. Table 1 assumes typical but generic values for the three estimated parameters, namely β_1=6, β_2=6, and G_{max}=50 m^2/ha, and employs West's (2004, p.42) generic under-bark volume equation $V_u=0.281D^{1.91}H^{1.02}$ and Shiver and Brister's (1992) conversion equation $V_d=V_u(1-d^{3.4138}D^{-3.3125})$ where d is the merchantable diameter limit (cm). Because the three equations are simple linear relationships, it is unsurprising that an unconstrained optimization tended to seek high stockings to maximize biomass (Table 1). However, optimising for the maximum mean annual increment of sawlog volume (to 12 cm small end diameter) did not suffer the same limitation, and indicated a realistic practical thinning regime (Table 1; cf. Sutton, 1976; Harris 2007), despite the unrealistic assumption that all trees are identical to the mean tree.

Optimizing for discounted sawlog value required both a discount rate and a seedling cost to avoid unrealistically high stocking and rotation lengths. The discount rate was set at 10%, seedlings were assumed to cost $0.1, and the price premium for large sawlogs was assumed to be the lesser of 4 and P=0.15D-1.5, based on the relative sawlog price data tabulated by Henson and Vanclay (2004). Given this discount rate and these prices, the optimal stocking regimes seem reasonable and practical.

It is not intended that Table 1 should form the basis for a silvicultural prescription for a hypothetical species. Table 1 was intended to demonstrate that the three equations behave reasonably, even when extrapolated to extreme situations, and when exposed to optimization routines able to reveal imperfections in models.

Figure 5 offers a further demonstration of utility by comparing West and Mattay's (1993) model $\ln(V)=b_1+b_2/t+b_3S$ (where t is age in years and S is site index) fitted to the same data using a traditional statistical approach with the equivalent prediction prepared using equations 1-3, calibrated to the mean values observed in the present analysis (Table 2). Parameters β_1 and β_2 are simple averages observed in the data; G_{max} and the small end diameter d are arbitrary estimates based on visual inspection of the data, and the initial stocking N_0 was estimated by least squares to provide a good fit. The pairs of lines for both *E. pilularis* and *E. grandis* are similar (Figure 5), offering reassurance that the three equations (1-3) are of practical relevance.

Synthesis and Application

The three relationships described above allow the calibration of 'rules of thumb' for the prediction of stand height, tree diameters and stem numbers, from minimal field data. In the limiting case, these relationships can be calibrated from a single observation of age, height, diameter and stocking taken from a temporary plot on a single occasion. These relationships are easily calibrated, and can be implemented as a compact computer-based spreadsheet accessible to many users. These approximations are not intended as a substitute for carefully formulated, calibrated and tested custom equations, but may provide a useful first approximation in situations where data are scarce.

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Table 1. Optimization results, indicating stocking and thinning regimes that arise from the application of equations 1-3.

Objective	Thinning	Planted stems/ha	Total Volume (incl thinnings)	Clearfall age	Mean dbh at clearfall	Thinning age	Residual stems/ha
Biomass MAI	0	10000	255	5	8		
	1	No benefit over direct regime					
Sawlog MAI (to 12 cm sed)	0	3740	315	15	18		
	1	8460	491	17	19	11	2880
	2	No benefit over single thinning					
Discounted value of sawlogs (10%)	0	280	245	24	31		
	1	560	211	30	36	11	100
	2	1300	374	34	36	11	970
						15	100

Table 2. Parameters used to compare West and Mattay (1993) model with present equations 1-3.

Parameter	*E. pilularis*	*E.grandis*
West and Mattay (1993) model		
b_1	3.61	2.75
b_2	-28.4	-42.5
b_3	0.093	0.148
Top height, S (m at age 20)	47	34
Equations 1-3, this study		
β_1	5.5	7.9
β_2	5.6	4.9
G_{max} (m^2/ha)	95	100
d, small end diameter (cm)	10	15
N_0, initial stocking (stems/ha)	4700	1110

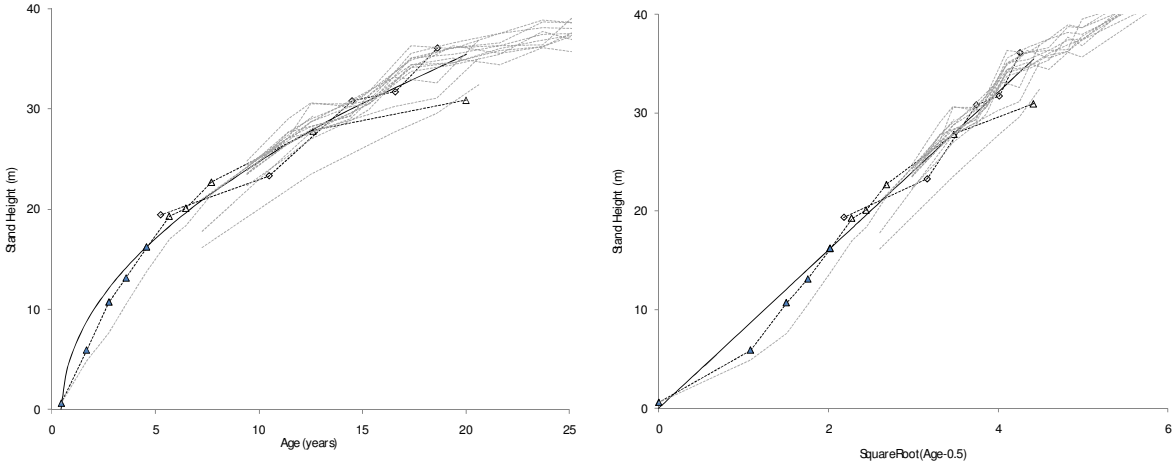

Figure 1. Plot 75 shown as diamonds; plot 88 shown as triangles. Solid line is an extrapolation based equation (1) calibrated for plot 88 at age 4.5. Grey lines illustrate are other *E. grandis* and *E. pilularis* plots with 8 or more measures and first measure at age 10 years or less. Right pane illustrates how the square-root transform linearizes the growth trend.

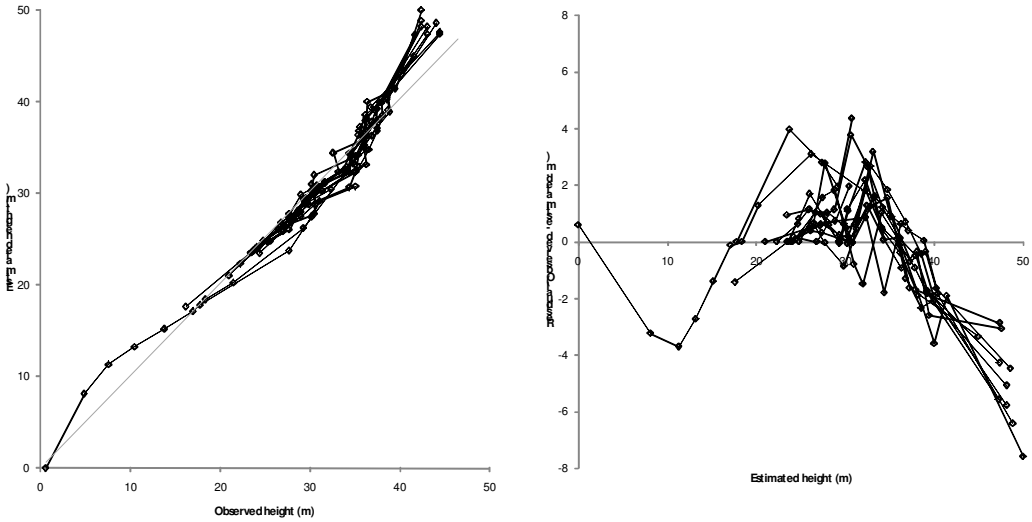

Figure 2. Tree heights observed and estimated from equation (1) calibrated for each plot at the first available height measure after age 5. In the left pane, the diagonal line $H_{est}=1.004H_{obs}$, indicates a small bias. The right pane shows residuals revealing a systematic bias.

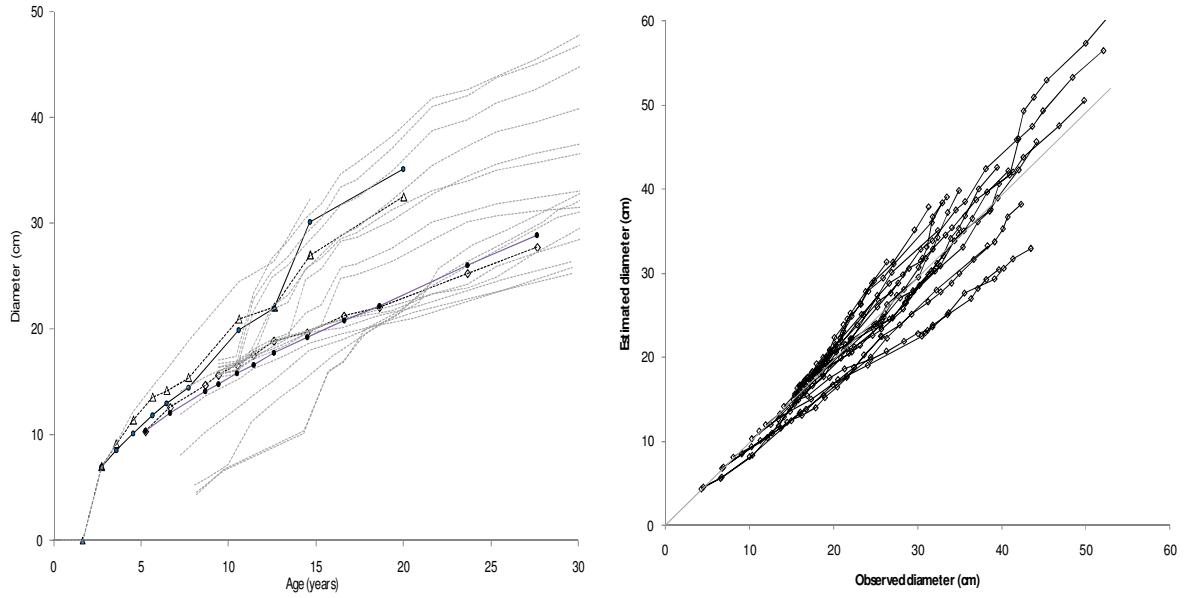

Figure 3. Observed and predicted growth trends. (a) Left pane illustrates Plot 75 as diamonds; plot 88 as triangles. Solid lines reflect predictions based equations 1 and 2 calibrated at age 4.5 (Plot 88) and 10.5 (Plot 75). Grey lines illustrate are other *E. grandis* and *E. pilularis* plots with 8 or more measures and first measure at age 10 years or less. (b) Right pane contrasts observations and predictions for all plots. Diagonal line is $D_{est}=0.983D_{obs}$, indicating a small bias.

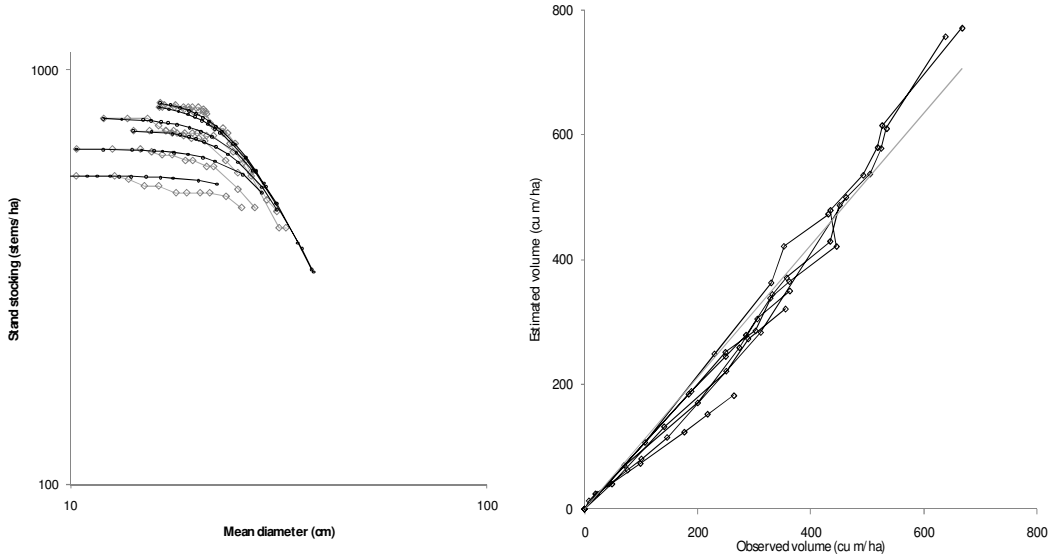

Figure 4. (a) Left pane shows diameter-stocking trajectories for observed (grey diamonds) and simulated (black circles) data. (b) Right pane contrasts observed and simulated stand volume (m^3/ha) trajectories (unthinned plots only). Diagonal line represents G_{est}=1.057G_{obs} indicative of a tendency to overestimate by 6%.

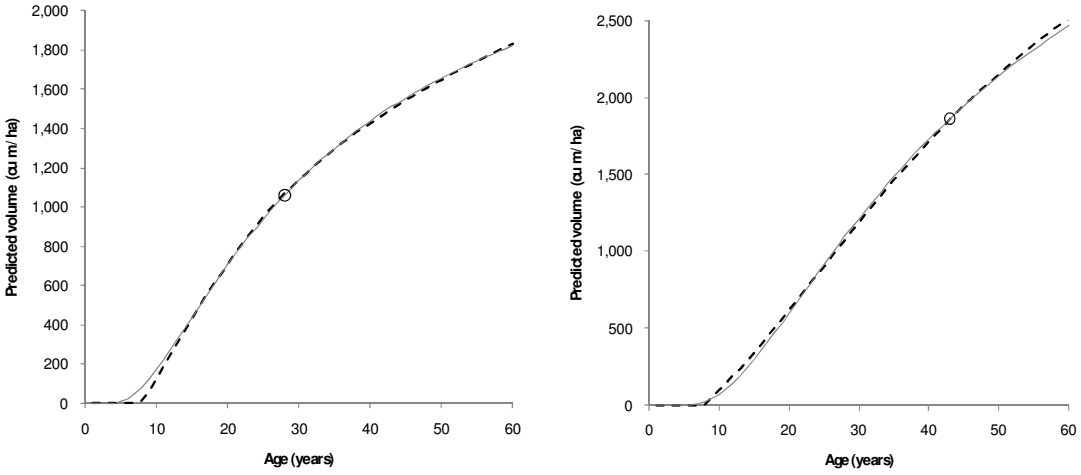

Figure 5. Predictions using equations 1-3 (dashed black line) contrasted with predictions by West and Mattay (1993; continuous grey line) for *E. pilularis* (left) and *E. grandis* (right). Symbol represents the age when mean annual volume increment is maximized, one possible indicator of rotation length.