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REFINED PLASTICITY MODEL FOR CONCRETE STRESS-STRAIN RELATIONSHIP PART I: PREDICTION OF PEAK STRESS AND RESIDUAL STRESS

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ABSTRACT

A refined plasticity model for concrete stress-strain relationships is proposed. The proposed failure surface has the ability to evolve its form based on empirical formulation in which being extracted from the experimental results via the frictional driver parameter (α). Two main features are highlighted in this paper such as the peak stress prediction and residual stress prediction of the proposed model. In this paper the comparison of proposed models with experimental results weighted on uniaxial-triaxial compression in axial direction. In the next part of the research a non-associative flow rule in which has an inclusion of size effect to be applied in the constitutive driver is proposed and experimental comparison in both axial and lateral direction is discussed.

KEYWORDS

Plasticity model, concrete stress-strain, triaxial-compression, peak stress, residual stress.

INTRODUCTION

Plasticity model is one of the most widely used method in predicting the stress-strain in concrete material under complex state of stress. One of the most important aspect in modelling using plasticity model is relied on the calibration with experimental results. The parameter that is used in the yielding function and also flow rule must be sensitive enough to be able to take account the characteristic behaviour of concrete material in uniaxial, biaxial and triaxial state. In this research the improvement over the one of the most cited yielding function of (Menetrey and Willam 1995) is discussed. New parameter in which it is called frictional driver parameter (α) is introduced. By using this parameter a suitable failure surface in which derived from empirical equation in peak axial level and also in residual axial level can be accommodated automatically. This will gives a different view of extending plasticity model to a new material by only feeding the empirical equation that have been calibrated with experimental results in both peak and residual stress level. Some comparison with experimental results (e.g. Dahl 1992; Hurlbut 1984; Xie, Elwi et al. 1995; Attard and Setunge 1996) that cover the wide range of concrete compressive strength in uniaxial and triaxial compression is shown and discussed. The proposed flow rule will be discussed in detail in the next part.

YIELDING FUNCTION, HARDENING AND SOFTENING LAWS

Yielding Function

The modification of yielding function of (Menetrey and Willam 1992) in which consist of additional frictional driver parameter has a general form as follows:

$$f = \left(\sqrt{1.5}\rho\right)^2 + q_h(k).m.\alpha\left[\frac{\rho}{\sqrt{6}}.r(\theta,e) + \frac{\xi}{\sqrt{3}}\right] - q_h(k).q_s(k) \le 0 \tag{1}$$

Where ξ, ρ, r, m, θ, e, q_h, q_s and α are hydrostatic length, deviatoric length, elliptic function, frictional parameter, lode angle, eccentricity of roundness, hardening parameter, softening parameter, parameter that controlling the peak stress and residual stress. Some of above expressions in detail are

$$\xi = \frac{I_1}{\sqrt{3}f_c}, I_1 = \sigma_1 + \sigma_2 + \sigma_3 \tag{2}$$

$$\rho = \frac{\sqrt{2J_2}}{f_c}, J_2 = \frac{1}{2}s_{ij}s_{ij}, J_2 = \frac{1}{6}\left[(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2\right] \tag{3}$$

$$m = 3\frac{f_c^2 - f_t^2}{f_c f_t}\frac{e}{e+1}, r(\theta,e) = \frac{4\left(1-e^2\right)\cos^2\theta + (2e-1)^2}{2\left(1-e^2\right)\cos\theta + (2e-1)\left[4\left(1-e^2\right)\cos^2\theta + 5e^2 - 4e\right]^{0.5}} \tag{4}$$

Where I_1, J_2, s_{ij}, σ_1, σ_2, σ_3, f_c and f_t are first invariant of stresses, second invariant of deviatoric stresses, component of deviatoric stresses, 1st principal stress (lateral pressure), 2nd principal stress (lateral pressure), 3rd principal stress (axial pressure), concrete compressive strength and concrete tensile strength.

The value of f_t used herein is not the common value adopted by several researcher (e.g. Papanikolaou and Kappos 2007; Bao, Long et al. 2013) where it is equal to 0.1 f_c or f_c/f_t=10. The value of f_t will be based on the works of (Attard and Setunge 1996) where two equation for concrete with silica fume and without silica fume is adopted and has a general form as follows:

$f_t = 0.9f_{sp} = 0.9\left(0.32(f_c)^{0.67}\right)$, Without silica fume, for f_c < 50 MPa (5)

$f_t = 0.9f_{sp} = 0.9\left(0.62(f_c)^{0.5}\right)$, With silica fume, for f_c > 50 MPa (6)

The frictional driver parameter (α) that has been introduced before consist of a function that control the frictional parameter (m) in peak stress level where here will be denoted as α_{peak} and in residual stress level where here will be denoted as α_{res}. The empirical formulation for peak stress is based on (Attard and Setunge 1996) and it has a general form as follows:

$$f_{cc} = f_c\left(\frac{f_r}{f_t}+1\right)^{k_1}, k_1 = 1.25\left[1+0.062\frac{f_r}{f_c}\right](f_c)^{-0.21} \tag{7}$$

Where f_{cc}, f_r and k_1 are peak axial stress, confinement pressure and parameter of peak axial stress, respectively. In residual stress level the empirical formulation is based on (Samani and Attard 2012) and it has a general form as follows:

$$\frac{f_{res}}{f_{cc}} = 1 - \frac{1}{a_1\left(\frac{f_r}{f_c}\right)^{k_2}+1}, a_1 = 795.7 - 3.291f_c^{'}, k_2 = \left(5.79\left(\frac{f_r}{f_c}\right)^{0.694}+1.301\right) \tag{8}$$

Where f_{res}, a, k_2 are the axial stress at residual level and parameters of axial stress in residual stresses. In order to implement the empirical formulation into the yielding function via the frictional driver parameter (α), two set of equation that related to boundary condition at peak axial level and residual level is required. The first equation can be defined by setting the failure criteria of yielding function at the peak stress where $\sigma_1 = \sigma_2 = \sigma_{lat}$, $\sigma_3 = f_{cc}$ and $q_h(k_{peak}) = 1$, $q_s(k_{peak}) = 1$. The second equation can be defined by setting the failure criteria of yielding function at residual stress where $\sigma_1 = \sigma_2 = \sigma_{lat}$, $\sigma_3 = f_{res}$ and $q_h(k_{res}) = 1$, $q_s(k_{res}) = 0$. By using the boundary condition into Eq.(1) we have:

$$\left(\sqrt{1.5}\rho_{peak}\right)^2 + m.\alpha_{peak}\left[\frac{\rho_{peak}}{\sqrt{6}} + \frac{\xi_{peak}}{\sqrt{3}}\right] - 1 = 0 \tag{9}$$

$$\left(\sqrt{1.5}\rho_{res}\right)^2 + m.\alpha_{res}\left[\frac{\rho_{res}}{\sqrt{6}} + \frac{\xi_{res}}{\sqrt{3}}\right] = 0 \tag{10}$$

By solving individually of Eq.(9) and Eq.(10) we have

$$\alpha_{peak} = \frac{1-\left(\sqrt{1.5}\rho_{peak}\right)^2}{m\left[\frac{\rho_{peak}}{\sqrt{6}} + \frac{\xi_{peak}}{\sqrt{3}}\right]} \text{ , } \alpha_{res} = \frac{-\left(\sqrt{1.5}\rho_{res}\right)^2}{m\left[\frac{\rho_{res}}{\sqrt{6}} + \frac{\xi_{res}}{\sqrt{3}}\right]} \tag{11}$$

As mentioned earlier in order to be applied at different stages this two value need to be combined together when implemented into the yielding function. Thus resulting in proposed equation for frictional driver parameter (α) are as follows:

$$\alpha = \left(\alpha_{peak} q_s(k) + \alpha_{res}\left(1 - q_s(k)\right)\right) \tag{12}$$

From Eqn (12) if k is equal to 1 (at peak) then $q_s(k) = 1$ this condition will gives the value of α equal to α_{peak} and if k is equal to ∞ (at residual) then $q_s(k) = 0$ this condition will gives the value of α equal to α_{res}. When k is between $1 < k < \infty$ the value of α will be somewhere between α_{peak} and α_{res}. The effect of frictional driver parameter (α) into the yielding function in compression meridian can be seen in Figure 1.

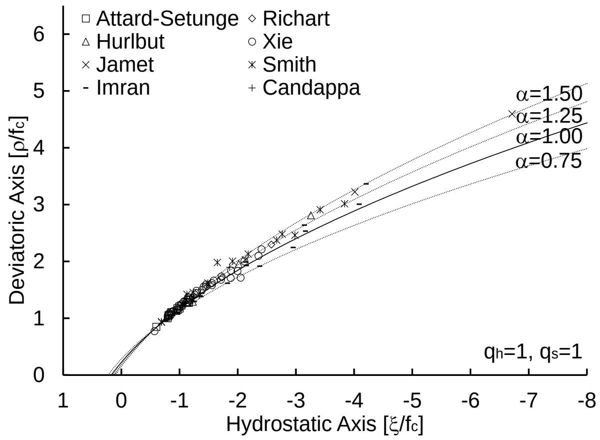


Figure 1. Effect of frictional driver parameter in yielding function at compression meridian section

Hardening and Softening Laws

Hardening parameter (q_h) is based on the proposed equation by (Carrazedo, Mirmiran et al. 2013) where the general expression is rewritten here as:

$$q_h(k) = \sigma_{cr} + \frac{\left[1-\sigma_{cr}\right].r_1.k}{r_1 - 1 + k^{r_1}} \tag{13}$$

$$k = \frac{\varepsilon_v^p}{\varepsilon_{v,\text{peak}}^p} \text{ , } r_1 = \frac{E_a}{E_a - E_b} \text{ , } E_b = 1 - \sigma_{cr} \text{ , } \sigma_{cr} = \left(\frac{\sigma_{3pr}}{f_c'}\right)^2 \tag{14}$$

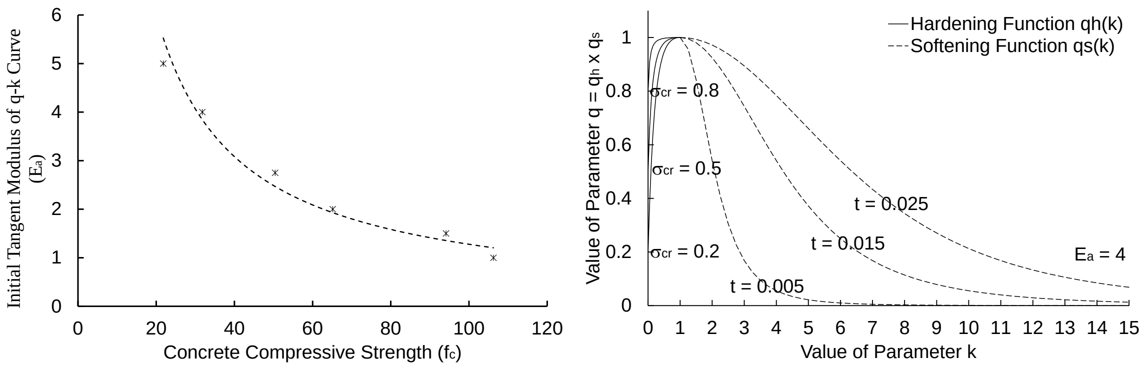
Where ε_v^p, $\varepsilon_{v,peak}^p$, k, E_a, σ_{cr} and σ_{3pr} are the plastic volumetric strain at specific incremental, plastic volumetric strain at peak in uniaxial compression or triaxial compression, ratio that defined the non-dimensional plastic volumetric incremental, initial tangent modulus of q-k curve in hardening function, critical stress that is based on the behavior under uniaxial compression, normalized parameter of stress that represent the elastic limit under uniaxial compression (usually varies from 0.3 f_c to 0.45 f_c), respectively. Both E_b and r_1 are parameters of ascending hardening function.

From the experimental results conducted by (Dahl 1992) the relationship between the concrete compressive strength (f_c) and E_a is shown in Figure 2a where it can be expressed in equation as $E_a=107.01(f_c)^{-0.962}$. For simplicity in general purpose modeling of stress-strain relationship the value of E_a can be set to 4.

The softening function used herein is derived from previous researcher (e.g. Bao, Long et al. 2013 ; Papanikolaou and Kappos 2007), the general expression are

$$q_s(k) = \left(\frac{t^2}{t^2 + {\varepsilon_{v,\mathrm{peak}}^{p}}^2 (k-1)^2} \right)^2 \tag{15}$$

Where $\varepsilon_v{}^p{}_{,\mathrm{peak}}$ and t are the plastic volumetric strain at the peak and parameter that control the rate of softening function. The value of parameter t will be discussed more detail on implementation of size effect and it is differ with the proposed equation by (Papanikolaou and Kappos 2007). The effect of hardening and softening function on the shape of q-k curve are shown in Figure 2b.

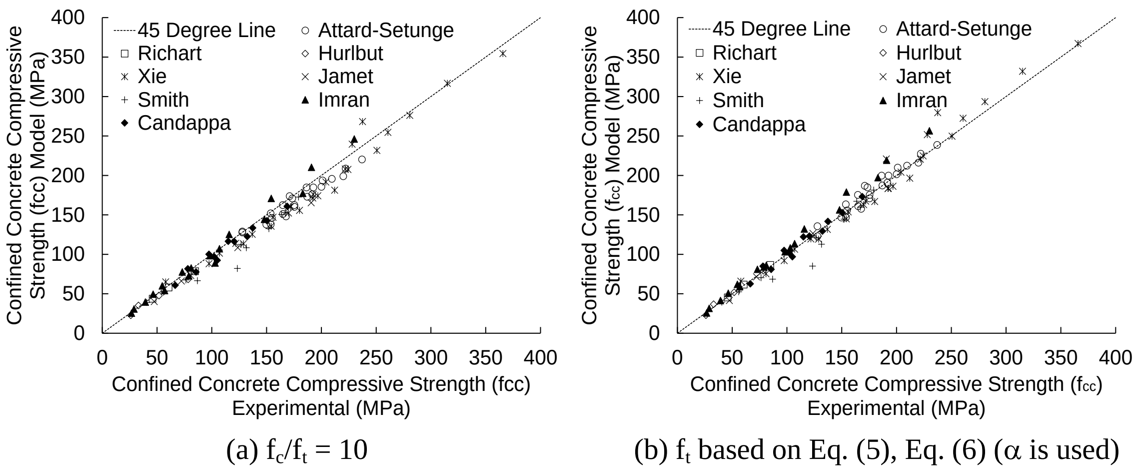


(a) Relationship of Ea with fc (b) Relationship of q with k

Figure 2. Relationship of parameter in hardening and softening function

PEAK STRESS AND RESIDUAL STRESS COMPARISON

In this section the comparison of peak stress and residual stress with experimental results from the original yielding function (Menetrey and Willam 1995) and the proposed yielding function with frictional driver parameter is shown in Figure 3. It can be inferred from Figure 3 that the proposed yielding function perform better in predicting the peak axial stress in concrete with active confinement pressure compared with the original yielding function that shown an underestimate as the confinement is increasing.



(a) $f_c/f_t = 10$ (b) f_t based on Eq. (5), Eq. (6) (α is used)

Figure 3. Comparison of peak axial stress prediction of original model with different ratio of f_t/f_c

The comparison of residual stress between experimental results, original yielding function and proposed yielding function is shown in Figure 4. It can be seen from Figure 4 that as the confinement stress is increasing the difference between residual stress and peak axial stress is become smaller. While both yielding function is able to predict this but the experimental results shown that at low confinement there is quite a large scatter of data due to differences in concrete compressive strength. It can be infered from Figure 4 that the original yielding function is not entirely suitable to predict the residual stress. On the other hand the proposed yielding function in which has a frictional driver parameter (α) able to outperform the original model in predicting the residual stresses.

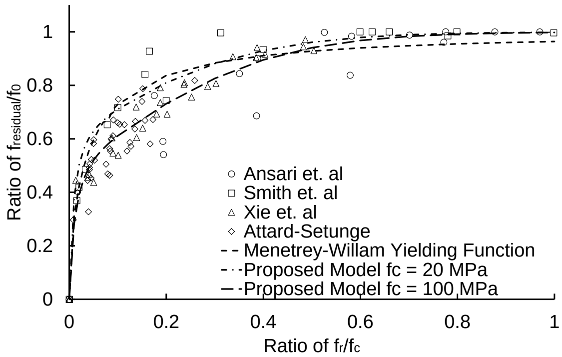


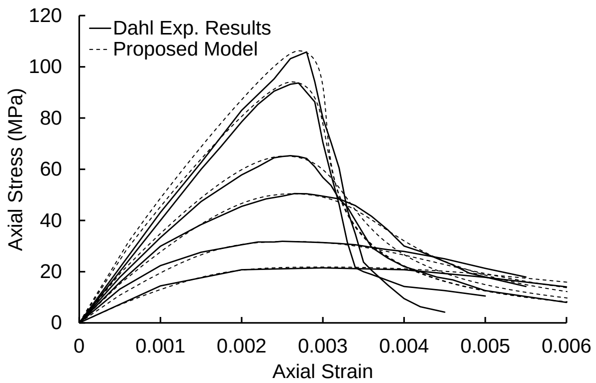
Figure 4. Comparison of residual stress prediction (α is used)

EXPERIMENTAL COMPARISON

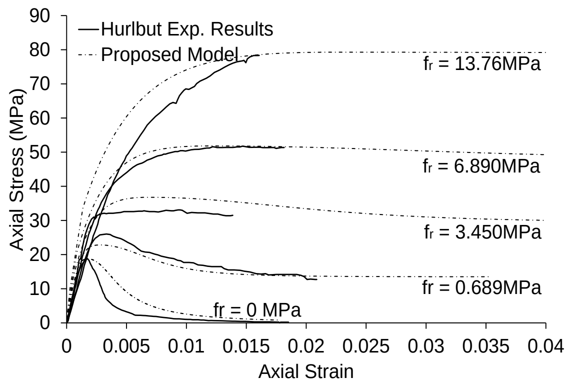
Even though the proposed flow rule is not yet discussed in this paper (it will be discussed on the next part) some comparison with experimental results is discussed to get a better understanding and also another different point of view of the importance in having a better prediction of peak axial stress and residual stresses. Some input parameter regarding with the proposed plasticity model is shown in Table 1 (The value of t will be discussed in the next part). The comparison with experimental results (e.g. Dahl 1992; Hurlbut 1985; Xie, Elwi et al. 1995; Attard and Setunge 1996) are shown in Figure 5 and Figure 6. It can be seen from Figure 5 and Figure 6 that the proposed plasticity model is able to give a good prediction of stress strain relationships in wide range of concrete compressive strength (fc) from 18.76 MPa to 132 MPa and also with various active confinement pressure from 0 to 0.5 f_r/f_c.

Table 1. Input General Model Parameter in Comparison with Experimental Results

Researcher	f_c (MPa)	ρ (kg/m^3)	E_c (MPa)	f_r (MPa)	E_a	$t_{uniaxial}, t_{triaxial}$	d, h (mm)	Silica Fume
Dahl	21.80	1710	14196	0	5.52	0.022272,-	100,200	No
	31.79	2000	21684		3.89	0.014832,-		No
	50.43	2150	30441		2.46	0.010518,-		Yes
	65.16	2300	38286		1.93	0.008266,-		Yes
	94.14	2400	49053		1.35	0.006377,-		Yes
	106.3	2400	52125		1.20	0.006001,-		Yes
Attard & Setunge	132.0	2250	43020	5,10,15	4	0.0058,0.0163	100,200	Yes
Xie et. al	92.00	1800	31497	0~44.5	4	0.0101,0.0196	55.5,110	Yes
Hurlbut	18.76	2600	21897	0~13.76	4	0.0143,0.0064	54,108	No

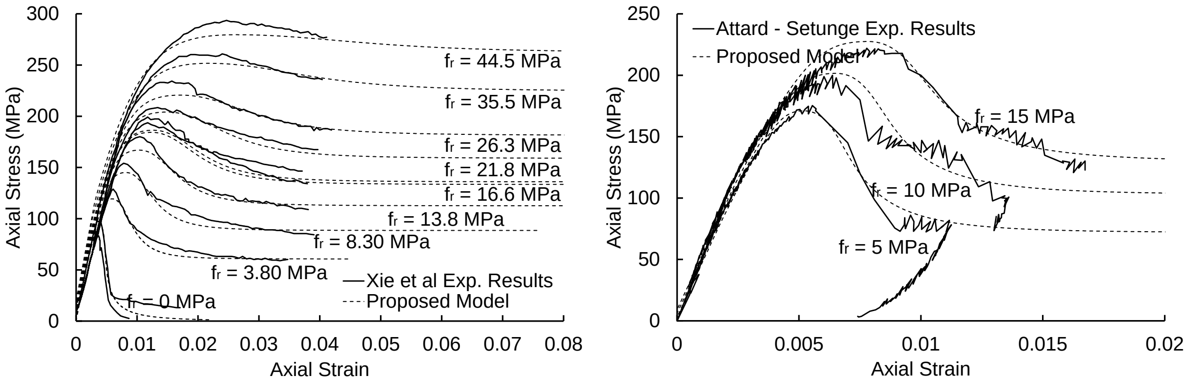


(a) (Dahl 1992)



(b) (Hurlbut 1985)

Figure 5. Comparison of proposed model with experimental results



(a) (Xie, Elwi et al. 1995) (b) (Attard and Setunge 1996)

Figure 6. Comparison of proposed model with experimental results

CONCLUSION

A refined yielding function that has an implemented frictional driver parameter (α) is able to improve the prediction in both peak axial stress and residual stresses. The application is simple and robust without having any iteration as a results of direct solution by using boundary condition at peak level and residual level. The comparison on stress-strain relationship using proposed model is shown very satisfactorily that cover a wide range of concrete compressive strength (18.76 MPa ~ 132 MPa) and also with various level of confinement (0 ~ 0.5 f_r/f_c). One of the other advantages of having the frictional driver parameter (α) we can use a different failure surface from empirical model and used it as an input into proposed model where subsequently the failure surface will changed. This will be very helpful in the future research when we want to extend to FRP or concrete with fiber.

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