Service-load modelling of deconstructable composite beams with friction-grip bolted shear connection

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SERVICE-LOAD MODELLING OF DECONSTRUCTABLE COMPOSITE BEAMS WITH FRICTION-GRIP BOLTED SHEAR CONNECTION

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ABSTRACT

Many new medium-sized office building structures are “temporary” and in some cases are demolished within ten years of their construction. Composite steel-concrete structural systems are very popular for medium-sized office buildings, but deconstruction and re-use in deference to demolition is difficult because headed stud shear connectors in the composite slab and steel beam flooring system cannot be detached easily, and reuse is virtually impossible. As an alternative, it is proposed that precast concrete panels can be attached to steel beams using pre-tensioned bolts, instead of cast-in-situ floors with pre-welded headed stud connectors. The proposed floor system can be deconstructed by unbolting the precast panels, enabling recyclability of the system and providing significant advantages in a paradigm within the construction industry focussed on low emissions. This paper presents an analysis of this slab and beam system, with the composite beams comprising of two elastic elements connected at their interface by a stiff but finite frictional connection. Under loading, it is shown that three distinct lengthwise domains exist along the member, and a model for this is presented and discussed.

KEYWORDS

Deconstructability, composite beams, friction-grip bolting, partial interaction, precast

INTRODUCTION

Most new medium-sized office building structures have a somewhat short lifespan. With composite steel-concrete structural systems being very popular in constructing medium-sized office buildings, deconstruction and re-use in deference to demolition is difficult because headed stud shear connectors in the composite slab and steel beam flooring system cannot be detached easily, and reuse is virtually impossible. As an alternative, it is proposed that precast concrete panels can be attached to steel beams using friction-grip or pretensioned bolts, instead of cast-in-situ floors with pre-welded headed stud connectors. The proposed demountable floor system can be deconstructed by unbolting the precast panels, enabling recyclability of the system and providing significant advantages in a paradigm in the construction industry focussed on low emissions (Bradford and Pi 2012, Bradford 2013).

An analysis of this slab and beam system is presented herein, with the composite beam comprising of two elastic elements connected at their interface by a stiff frictional connection provided by the pretensioned bolts whose response can be considered as being rigid-plastic. It is shown that relatively low pretension forces are needed to produce a very stiff beam with close to full interaction throughout the service load range of a typical composite beam.

ANALYSIS

A simply supported prismatic composite beam is considered to comprise of two elements: the top (Ωc) of area Ac and bottom (Ωs) of area As joined by pre-tensioned high-strength bolts tensioned and...
distributed uniformly along the member of length $2L$. The beam is further assumed to be subjected to a uniformly distributed load $q$, with the pre-tension providing a friction at the interface between $\Omega_i$ and $\Omega_{b}$ distributed uniformly of magnitude $f_i$. Pragmatically, the bolts are considered to be placed in clearance holes rather than being fitted bolts (Trahair et al. 2008), with a potential to slip on average by a value $s_0$ determined based on the diameters of the bolts and the clearance holes. Under monotonically increasing load $q$ starting from $q = 0$, a shear flow force develops at the interface, being given by

$$f = qxQ/I,$$

(1)

in which $x$ is measured from the origin located at mid-span, $Q$ the first moment of area of region $\Omega$, about the elastic centroid of the cross-section denoted $\Omega = \Omega_i \cup \Omega_b$ and $I$ the second moment of $\Omega$ about this axis. The maximum value of the shear flow force $\pm qxQ/I$ is at the ends of the member, and so assuming the frictional force prevents any slip, the beam will have full interaction provided $q \leq q_1 = fIQL$. When $q > q_1$, slip takes place in the regions that flank the beam until a value of $q = q_2$ is attained, and the slip $s$ at these flanking regions ($s = s_0$) is sufficient for the end bolts to bear against their clearance holes. Further loading then defines three lengthwise domains along the beam as $\Gamma_1$: $x \in [-a, a]$; $\Gamma_2$: $x \in [-b, -a] \cup [a, b]$ and $\Gamma_3$: $x \in [-L, -b] \cup [b, L]$. These domains are depicted in Figure 1.

![Figure 1. Regions of different shear connection](image)

The differential equation of bending is

$$d^3v/dx^2 = v_x = q\left(x^2 - L^2\right)/2\psi_i,$$

(2)

in which $v$ is the transverse deformation of the beam and $\psi_i$ the flexural rigidity appropriate to the relevant domain $\Gamma_i$ ($i = 1, 2, 3$). As a sensible approximation and based on push test results (Lee and Bradford 2013), it is assumed that $\psi_1 = \psi_3 = EI$ and $\psi_2 = EI_b$, where $EI_b$ is the flexural rigidity of the counterpart non-composite beam. A more refined model would, of course, be based on elastic partial interaction shear stiffnesses of $k_i$ within each $\Gamma_i$ and the current model uses the special case that $k_1 = k_3 = \infty$ and $k_2 = 0$.

Integrating Eq. 2 within the domain $\psi_i$ produces

$$v = \frac{qL^2}{24\psi_i} (x^2 - 6) + LK_{b_i}x + K_{b_i}, \quad \text{and} \quad v' = \frac{qL^3}{6\psi_i} x^3 + K_{b_i},$$

(3)

where $K_{b_i}$ and $K_{h_i}$ are constants of integration relevant for each region $\Gamma_i$ and $x = x/L$. Specifically,

$$v = \frac{qL^4}{24\psi_i} (x^2 - 6) + C_2 \quad \text{and} \quad v' = \frac{qL^3}{6\psi_i} x^3 - 3$$

(4)

for the region $\Gamma_1$ when the symmetry condition $v'(0) = dv(0)/dx = 0$ is invoked;

$$v = \frac{qL^4}{24\psi_i} (x^2 - 6) + C_4 \quad \text{and} \quad v' = \frac{qL^3}{6\psi_i} x^3 + C_3$$

(5)

$$v = \frac{qL^4}{24\psi_i} (x^2 - 6) + C_6 \quad \text{and} \quad v' = \frac{qL^3}{6\psi_i} x^3 + C_3$$

(6)

for the regions $\Gamma_2$ and $\Gamma_3$ respectively, in which $C_2, \ldots, C_6$ are constants of integration. These may be found by imposing the kinematic conditions and continuity of deflection and slope as

$$C_2 = \frac{qL^4}{24\psi_i} \left[5 + (1 - n) \left(3\alpha^3 - 3\beta^3 - 8\alpha^2 - 8\beta^2 + 3\alpha + 3\beta - 12\right) \beta \right],$$

(7)
\[ C_3 = \frac{qL^3}{6\psi_1} (1-n)\alpha(\alpha^2 - 3), \]  
\[ C_4 = \frac{qL^4}{24\psi_1} \left( 5 + (1-n)\beta^2 \left( \beta^2 - 6 \right) + 4(1-n)\alpha (\beta^2 - 1) \right), \]  
\[ C_5 = \frac{qL^4}{6\psi_1} (n-1)\alpha(\beta^2 - 3) - \alpha(\alpha^2 - 3), \]  
\[ C_6 = \frac{qL^4}{24\psi_1} \left[ 5 + 4(1-n)\alpha(\beta^2 - 3) - \alpha(\alpha^2 - 3) \right]. \]

For the portion of the beam having full interaction, the axial force in the steel is

\[ N_s = \frac{A_s y_s M}{I} = \frac{A_s y_s qL^2}{2I} (1 - \xi^2) = A_s E_s u_s', \]

where \( E_s \) is the elastic modulus of the steel section, and \( y_s \) the (positive) distance to its centroid from that of the transformed cross-section. Solving Eq. 12 subject to the symmetry condition \( u_s(0) = 0 \) gives

\[ u_s = \frac{qL^3 y_s}{6E_s I} (3 - \xi^2), \]

and specifically at the end of the region of full interaction defined by \( x = a \),

\[ u_s(\alpha) = \frac{qL^3 y_s}{6E_s I} (3 - \alpha^2). \]

By a similar argument for the concrete slab, it can be concluded that

\[ u_c(\alpha) = \frac{qL^3 y_c}{6E_c I} (3 - \alpha^2). \]

Figure 2. Free body diagram showing horizontal forces at end of beam elements

The slip deformation \( s \) at the interface between the concrete slab and steel beam is

\[ s = u_s - u_c + h\nu', \]

where \( h = y_s - y_c \) is the distance between the element centroids, and for the case that \( \alpha = 1 \) and the beam deflections are governed by Eq. 4, using Eqs. 4, 13 and 16 produces \( s = 0 \) as expected. Figure 2 shows a free body diagram of the concrete and steel elements for \( q > q_1 \) (i.e. \( \alpha < 1 \)), for which the sliding friction is taken as \( f_i \) and for which the bolts have not commenced to bear in their clearance holes (\( \beta = 1 \)). For the steel,

\[ N_s = f_i (L-x) = A_s E_s u_s', \]

which can be integrated subject to the boundary condition given in Eq. 14 to yield

\[ u_s = \frac{f_i L^2}{2A_s E_s} \left[ \xi (2 - \xi) - \alpha (2 - \alpha) \right] + \frac{qL^3 y_s}{6E_s I} (3 - \alpha^2), \]

while for the concrete,

\[ N_c = f_i (L-x) = -A_c E_c u_c'. \]
which produces
\[
\frac{qL^2}{2AE}\left[\alpha(2-\alpha)-\xi(2-\xi)\right]+\frac{qL^2y\gamma}{6EI_\nu}(3-\alpha^2).
\]
(20)
The slip can then be found from Eqs. 16, 5, 8, 19 and 20 as
\[
s = \frac{fL^2}{AE}\left[\xi(2-\xi)-\alpha(2-\alpha)\right]+\frac{qL^2h\mu}{6AE_\nu}\left[\xi(\xi^2-3)\alpha(\alpha^2-3)\right],
\]
in which
\[
\frac{1}{AE} = \frac{1}{AE_0} + \frac{1}{AE_\nu}.
\]
(22)
Noting that \( q = fL/(\alpha QL) \) allows the slip in Eq. 21 to be written in the dimensionless form
\[
\frac{s}{L} = \left(\frac{fL}{AE}\right)\left[\xi(2-\xi)-\alpha(2-\alpha)\right]+\left(\frac{hAE}{EQ}\right)\left[\xi(\xi^2-3)\alpha(\alpha^2-3)\right],
\]
which identifies the significance of the dimensionless parameters \( fL/\sqrt{AE} \) and \( hAE/\sqrt{EQ} \). The load at which bearing in the clearance holes commences is determined when \( s = s_b \) and is at \( \xi = 1 \). Substituting these into Eq. 23 then produces
\[
(1-p)\alpha^3-2\alpha^2+(1-m+3p)\alpha-2p=0,
\]
(24)
which defines the location of the point at first slip \((\alpha L)\) as a function of the dimensionless variables
\[
m = \frac{AE_\nu s_b}{fL^2} \quad \text{and} \quad p = \frac{AEh}{EQ}.
\]
(25)
When \( s_b = 0 \), the bolts commence to bear adjacent to the location of first slip, and so the solution of Eq. 24 is \( \alpha = 1 \), while if the bolts are allowed to slip without restraint, \( m \to \infty \) and the solution of Eq. 24 is \( \alpha = 0 \), as expected. Following this, the location of the first bearing of the bolts can be determined from Eq. 23 by setting \( \xi = \beta \), producing the cubic equation
\[
p\beta^3-\alpha\beta^2+(2\alpha-3p)\beta+(1-p)\alpha^3-2\alpha^2+(3p-m)\alpha=0.
\]
(26)

![Figure 3. Plot of \( \beta \) against \( \alpha \) for \( p = 1.0 \)](image)

**ILLUSTRATION**

Figure 3 shows a family of curves of \( \beta \) against \( \alpha \) when \( p = 1.0 \), while Figure 4 gives the counterpart curves for \( p = 5.0 \). It can be seen that as the limiting value of the slip related to the size of the clearance hole relative to the bolt diameter \( s_b \) increases (and so \( -m \) increases), the difference between \( \alpha \) and \( \beta \) increases, indicating an increase in the region \( \Gamma_2 \) in which the bolts slip. An increase in the parameter \( p \), viz. an increase in the distance between the centroids of the elements \( h \), decreases the region \( \Gamma_2 \) slightly.
The central deflection \( v(0) \) is obtained from Eqs. 5 and 7 as
\[
v_0 = \frac{qL^4}{24\psi_1}\left[5 + (1-n)\left(3\alpha^3 - 4\alpha^2 - 6\alpha + 12\right)\alpha - \left(3\beta^3 - 4\beta^2 - 6\beta + 12\right)\beta\right],
\] (27)
which can be recast in dimensionless form by normalising with respect to the central deflection of a beam with flexural stiffness throughout \( 5qL^4/24\psi_1 \) as
\[
\omega = 1 + \frac{(1-n)}{5}\left[3\alpha^3 - 4\alpha^2 - 6\alpha + 12\right]\alpha - \left(3\beta^3 - 4\beta^2 - 6\beta + 12\right)\beta\right].
\] (28)
When the member has full interaction throughout \( i.e. f_i \to \infty \), \( \alpha = \beta = 1 \), and \( \omega = 1 \) in Eq. 28, while when it has no interaction (being imposed by \( f_i = 0 \)), \( \alpha = 0 \) and \( \beta = 1 \), and \( \omega = n \) in Eq. 28 as expected.

The evolution of the deformation of a composite beam is depicted in Figure 5, in which the dimensionless load \( q/q_1 = 1/\alpha \). For this beam, it is twice as stiff when composite compared with being non-composite \( (viz. n = 2.0) \), and the parameter \( p \) in Eq. 25 is taken as 1-0. The parameter \( m \) in Eq. 25 captures both the frictional resistance \( f_i \) and the limiting slip \( s_b \). When \( s_b = 0 \) and so \( p = 0 \), there is no region of slip and so the beam has full interaction throughout. However, as \( s_b \) increases, the beam has increasing regions of slip with no interaction, and hence the stiffness of the beam is reduced, as can be seen from Figure 5. The maxima of the curves correspond to the location of \( q_2 \) when the bolts commence to bear in their clearance holes.

A typical composite beam is considered with a slab width of 2000 mm and depth 150 mm, for which it is assumed that the steel section is an Australian 460UB82-1, that the elastic moduli of the steel and concrete are 200 GPa and 25 GPa respectively and that the simply supported member of length 2L = 9 m is subjected to a uniformly distributed load. Based on modular ratio theory, \( \psi_1 = 246.6 \times 10^6 \) kNmm, \( \psi_2 = 88.5 \times 10^6 \) kNmm and so \( n = 2.79 \). If the beam has 16 M20 bolts tensioned to 150 kN (which is typical), then \( f_i = 150 \times 16 \times 0.45 / 9000 = 120 \) N/mm, which assumes that the coefficient of friction is 0-45, as has been found in push testing (Lee and Bradford 2013). Also for this beam, \( I = 1233 \times 10^6 \) mm\(^4\) and \( Q = 2.5 \times 10^6 \) mm\(^3\), which leads to \( q_1 = 13.2 \) kN/m and which represents a load of 6-6 kPa. The deflection magnification for this beam is shown in Figure 6, in which it can be seen that the
Deflections increase above those for the counterpart beam will full interaction once the load $q_1$ is exceeded. However, when the load to cause first bearing is reached, the flanking regions of the beam enter a regime of full interaction again and the beam stiffens with an increase of load.

![Deflection magnification for composite beam](image)

**CONCLUSIONS**

This paper has proposed the use of bolted shear connectors in composite beams as a sustainable replacement of welded headed shear connectors, whose deconstructability and re-use is not possible. High-strength friction grip bolts are more expensive to manufacture and to install than are high-strength bolts, but when the cost of deconstructability and re-use, as well as the need for fewer high-strength bolts than headed connectors are taken into account, the technology shows considerable promise. The pre-tensioning delays the onset of interface slip, and has ramifications on the structural response at service load levels, as does the slip of the bolts in oversized clearance holes. These variables can be captured by a prescriptive equation that was developed in the paper.

**REFERENCES**


