Constitutive behaviour of flowing granular soils near walls

P Rognon  
*University of Sydney*

T Miller  
*University of Sydney*

I Einav  
*University of Sydney*

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CONSTITUTIVE BEHAVIOUR OF FLOWING GRANULAR SOILS NEAR WALLS

P. Rognon*
Particles and Grains Laboratory, School of Civil Engineering, The University of Sydney, NSW, 2006, Australia. pierre.rognon@sydney.edu.au (Corresponding Author)

T. Miller
Particles and Grains Laboratory, School of Civil Engineering, The University of Sydney, NSW, 2006, Australia. thomas.miller@sydney.edu.au

I. Einav
Particles and Grains Laboratory, School of Civil Engineering, The University of Sydney, NSW, 2006, Australia. itai.einav@sydney.edu.au

ABSTRACT

We investigate the effect of bounding walls on the ability of granular soils to flow. We present a set of experimental and numerical granular flows using a novel Stadium Shear Device and discrete element method simulations. Our focus is on the simple configuration of plane shear flow of 2D grains between two parallel walls, while we analyse the time-averaged velocity profiles within the flows.

Simulations and experiments consistently show that the visco-plastic constitutive behaviour of such granular flows is strongly affected by the presence of walls over relatively large distances. Near the walls, the material appears to flow more easily, forming a layer where the shear rate is up to 100% higher than the rate far away from the wall.

This effect of walls is important for geotechnical applications such as foundation design and landslide run-out prediction, where a comprehensive understanding of the mechanical interaction between granular-soils and structures is crucial.

KEYWORDS

Granular soils, post-failure behaviour, wall effects, stadium shear device, experiments.

INTRODUCTION

Granular soils fail under particular loading conditions. Predicting this failure is crucial in many geotechnical applications related to foundation design and slope stability analysis. The flow of soils is an obvious occurrence after the onset of geotechnical failures. Understanding such flows is critical for the prediction of the extent of sinking of overloaded foundations and the determination of how far landslide could travel (i.e., landslide run-outs).

In the past 20 years, great advances have been made to our understanding of the flowing behaviour of granular materials such as soils. Their constitutive behaviour was established, taking the form of a visco-plastic law (GDR 2004; Dacruz et al. 2005; Job et al. 2005, 2006; Forterre and Pouliquen 2008; Andreotti et al. 2012):

$$\tau = \mu(I) \sigma$$  \hspace{1cm} (1)
where $\tau$ and $\sigma$ are the shear and normal stresses, respectively; $\mu$ is a dynamic coefficient of friction, which is a function of the shear strain rate $\dot{\gamma}$ via the inertial number $I$:

$$\mu(I) = \mu_1 + (\mu_2 - \mu_1) / (1 + I_0 / I)$$  \hspace{1cm} (2a)

$$I = \dot{\gamma} t_i$$  \hspace{1cm} (2b)

The shape of the function (2a) covers friction coefficients between a minimum value $\mu = \mu_1$ for infinitely slow flow ($\dot{\gamma} \to 0$, where $\mu_1$ takes a typical value at the order of 0.35), and a maximum value of $\mu = \mu_2$ (where the order of $\mu_2$ is 0.65 for rapid flows, where $I \gg I_0$, with values of $I_0$ of the order of 0.3). For intermediate flow rates, $0 < I < I_0$, the friction coefficient increases with $I$, and thus with the shear rate, mostly linearly.

The inertial number $I$ is a dimensionless number comparing two times: the time associated to the shear, $\dot{\gamma}^{-1}$, and a time $t_i$ associated to local grain rearrangement. For dry granular materials without cohesion, featuring a narrow range of grain size $d$, this time corresponds to the inertial motion of a single grain, driven by the normal stress: $t_i = d \sqrt{\rho / \sigma}$, where $\rho$ is the grain density. For polydisperse mixture of grains of average size $D$, it was showed that the visco-plastic law (1) is still valid when introducing the size $D$ in the time scale $t_i$, such as $t_i = D \sqrt{\rho / \sigma}$ (Rognon et al. 2007). When grains are immersed in a viscous liquid, the law (1) was also showed to be valid when considering a time scale corresponding to a grain displacement driven by the confining effective stress $\sigma$, and hindered by the fluid viscosity $\eta$, $t_i = \eta / \sigma$ (Cassar et al 2005; Rognon and Gay 2009; Rognon et al. 2010a, 2011). Further extension of the visco-plastic law was explored for cohesive grains, affecting both $t_i$ and $\mu_1$ (Rognon et al. 2006, 2008).

The visco-plastic constitutive law is able to predict the flow of many granular soils far from walls. However, in many applications, soils flow near walls or solid boundaries. Understanding the effect of the wall on the granular soil’s ability to flow is therefore crucial. Yet, it is also a great challenge.

It was showed that smooth walls can lead to some sliding velocity, whereas a small wall roughness of the order of the grain size, or some rolling friction, leads to avoiding interfacial sliding (Shojaaee 2012a, 2012b). Further still, even without sliding, the presence of a wall may affect the granular soil ability to flow in its vicinity (Miller et al. 2013). Such a wall effect remains, however, poorly documented.

In this paper, we present a set of experimental and numerical results that exhibits a clear long-range effect of walls governing the visco-plastic behaviour of model granular flows. Experiences are based on the Stadium Shear Device (SSD) introduced in Miller et al. (2013), and the simulation method is the Discrete Element Method as introduced in Cundall and Strack (1979). Both experiments and simulations consider the simple flow configuration of steady shear between two parallel walls. The analysis will focus on the spatial distribution of the shear strain rates near the walls.

**EXPERIMENTAL GRANULAR FLOWS NEAR WALLS**

**Stadium Shear Device**

The SSD is an experimental apparatus that we specifically designed to produce plane shear flows of 2D granular materials (figure 1).

The SSD has been designed to test an assembly of plastic cylinders standing on a glass plate and sheared by a belt system (see figure 1a,b). Grains are 10mm high nylon cylinders of three different diameters (12, 15 and 20mm) to prevent crystallisation. The nylon-faced rubber belt has asperities that are semi-circular in cross-section with a 6mm radius and a centre-to-centre spacing of 14mm. These asperities ensure a non-slip boundary condition. Grains are sheared between two parallel sections of
the belt in the central region and recirculate underneath the sprockets at both ends. In the central region, the macroscopic shear rate $\dot{\Gamma}$ is controlled by the belt velocity $V_{\text{belt}}$, $\dot{\Gamma} = V_{\text{belt}} / H$ and the normal stress $\sigma$ by a system of constant force springs.

The advantage of this set-up is its ability to produce plane shear flows in the central region while the recirculation of the particles allows continuous shear for deformations as large as desired. The limitation of the set-up is its relative low number of grains in width, approximately 25.

Figure 1. Granular flows in the SSD. (a) Experimental device (total length of 1.5m); (b) pictures of the central region where grains are sheared between two parallel sections of the belt, producing a plane shear geometry where both shear strain rate $\dot{\Gamma} = V_{\text{belt}} / H$ and normal stresses $\sigma$ are prescribed. (c) Velocity profiles $V_x(y)$ and (d) shear rate profile $\dot{\gamma}(y) = \partial V_x(y) / \partial y$ in the transverse direction $y$.

Velocities and shear rates are normalized by the belt velocity and the macroscopic shear rate $\dot{\Gamma}$, respectively. In (c), the dashed line represents the linear velocity profile for comparison. (c,d) Symbols denote the profiles in flows of differing macroscopic shear rates $\dot{\Gamma}$, in the range 0.08 s$^{-1}$ to 1 s$^{-1}$. The confining pressure is $\sigma = 9.6$ kPa.

**Experiments**

In the following, we will present the results obtained from five flows of differing shear rates $\dot{\Gamma}$, in the range 0.1 s$^{-1}$ to 1 s$^{-1}$, and of similar normal stress $\sigma = 9.6$ kPa. Given the size and the density of grains, these flows correspond to an inertial number $I$ (see Eq. (2b)) ranging from $10^{-4}$ to $10^{-3}$.

Each flow is prepared by pre-shearing during a deformation of 100. Then, snapshots are taken in the central region (figure 1b), where the grains are sheared between two parallel sections of the belt. From
consecutive pictures, the trajectory of individual grains could be identified using a Particles Tacking algorithms based on Hough transform.

**Velocity and Shear Rate Profiles**

The velocity profiles $V_x(y)$ was deduced by averaging grain’s velocity $x$-component both in time and along the shear direction $x$. Figure 1c shows these profiles. When normalised by the belt velocity, they collapse on a single curve that is not linear, but exhibit some curvature.

The local shear rate profile $\dot{\gamma}(y) = \frac{\partial V_x(y)}{\partial y}$ was deduced by differentiating these profiles (see figure 1d). Consistently, it is not constant. There are higher values of shear rate near the wall than in the centre of the flow, with a factor of approximately 2 between the extremes.

Such a result is consistent with the results presented in Miller et al. (2013), using the same apparatus but focusing of flows with fixed solid fraction instead of fixed normal stress.

**Wall Effect**

In shear flows between two parallel walls, the conservation of momentum predicts that the stresses must be constant in the transverse direction $y$. If this was the case, the constitutive law Eq. (1) would predict a constant shear rate $\dot{\gamma}(y)$ and a linear velocity profile $V_x(y)$, which is not consistent with our results on figure 1a,b. Two possible explanations should be considered:

1. There are some long-range wall perturbations that modify the flowing behaviour of the material throughout the system.
2. There is some friction between the grains and the glass plate they stand on, which would lead to a non-uniform shear stress in the layer, being maximum near the wall and minimum in the centre.

Let us consider a numerical approach to discriminate between these two explanations.

**INSIGHTS FROM SIMULATIONS**

**Discrete Element Method**

We simulated the plane shear flow of a 2D granular material using a Discrete Element Method (see figure 2a). Grains are disks interacting through frictional and inelastic contacts. The grain-to-grain coefficient of friction was set to 0.5, and their coefficient of restitution to 0.8. The elastic repulsion is Hookean (linear spring), with a spring constant much larger that the scale of the normal stress, ensuring the grains to undergo small deformations. Walls are comprised of an array of grains moving in a synchronized fashion to produce the shear and the normal stress (see Rognon and Einav 2010b for more details).

The advantage of this simulation is that there is no friction between the grains and the glass plate.

**Numerical Experiment**

The numerical experiment starts by preparing a steady state flow. This is achieved by randomly setting the grains with no contact, and by applying the normal stress and the shear rate for a deformation of 100. The flows are then in a steady state regimes. Then, snapshots of grain velocities and positions are recorded every unit deformation during a total deformation of 100. Spatial and time averages are then performed from these data.
Velocity and Shear Rate Profiles

With the simulations, it is possible to measure the stresses within the flow (see for instance Rognon et al. 2010a). Figure 2b shows the profile of friction coefficient, ratio of normal to shear stress in a flow with $I=0.01$. The profile is constant, according to the prediction of the momentum conservation in absence of basal friction. Figure 1b also shows the profile of solid fraction, which is also constant excepted for small fluctuations near the walls.

Figure 2c shows the corresponding velocity and shear rate profiles. Likewise the experimental profiles, the velocity profile is not linear, and the shear rate profile is not constant. The variation in shear rate is also of the order of 2 between the highest shear rate near the walls and the lowest in the centre of the flow.

Wall Effect

This result clearly indicates that, even without basal friction, the granular soil ability to flow is modified near the wall. This perturbation is important, forming a layer near the walls of at least 10 grain width, with a high shear rate.

Figure 2. DEM simulation of plane-shear granular flow. (a) Simulated system: blue grains are flowing and black grains are the walls; the system is periodic in the x-direction. (b) Profile of dynamic friction coefficient (blue-bottom axis) and profile of solid fraction (red/top axis) in a flow with $I=0.01$. (c) Velocity (blue/bottom axis) and shear strain rate (red/top axis) profiles in the same flow.
CONCLUSIONS

This paper has presented a set of experimental and numerical results showing a strong effect of walls in the behaviour of granular soils flowing in their vicinity. This perturbation leads to a strong increase in shear strain rate near the wall, in a layer at least 10 grains width.

Such behaviour is important for application involving flows of granular soils near structures. However, it cannot be captured by the mere visco-plastic law. It seems that an additional model is required to comprehend this phenomena, that would predict the thickness of the high shear rate layer and how much higher the shear rate is in this layer, as a function of flow parameters and grains properties.

REFERENCES


