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TRANSIENT ANALYSIS OF WAVE PROPAGATION IN 3D SOIL BY USING THE SCALED BOUNDARY FINITE ELEMENT METHOD

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ABSTRACT

An efficient method for modelling the propagation of elastic waves in unbounded domains is developed. It is applicable to 3D soil-structure interaction problems involving scalar and vector waves. The scaled boundary finite element method is employed to derive an equation for the displacement unit-impulse response matrix on the near field / far field interface. The unbounded domain is divided into multiple subdomains where the displacement unit-impulse response matrices for individual subdomains are calculated separately and in parallel, leading to a reduction of computational effort. As the displacement unit-impulse response matrices approach zero, the convolution integral representing the force-displacement relationship can be truncated. After the truncation the computational effort only increases linearly with time. Thus, a considerable reduction of computational effort is also achieved by this temporal localization. In addition, a recursive algorithm for calculating the convolution integral based on a piece-wise linear variation of the displacement unit-impulse response matrix is developed, so that the time step size for transient analysis can be arbitrarily small without significantly affecting the total computational effort. Numerical examples demonstrate the accuracy and high efficiency of the proposed method.

KEYWORDS

3D soil-structure interaction, scaled boundary finite element method, displacement unit-impulse response matrix, convolution, truncation, multiple subdomains, recursive algorithm.

INTRODUCTION

The numerical modelling of wave propagation in unbounded domains is required in a number of engineering applications, such as soil-structure interaction analysis or dam-reservoir interaction analysis. Here, the well-established finite element method cannot be used straightforwardly, since outgoing waves are reflected at the artificial boundaries of the finite element mesh, such that special measures have to be taken to prevent these reflections (Givoli, 1999; Tsynkov, 1998). A popular method for the analysis of dynamic problems in unbounded media is the boundary element method, because the fundamental solution explicitly fulfils the radiation condition (Beskos, 1987, 1997). The idea of extending the finite element mesh towards infinity has driven the development of infinite element techniques (Bettes, 1992). A recent technique, which is particularly suitable for modelling...
time-dependent problems in infinite media, is the scaled boundary finite element method (SBFEM) (Wolf and Song, 1997). This semi-analytical technique is based on a combination of a numerical solution in the circumferential directions with an analytical solution in the direction of wave propagation. Thus, radiation damping is modelled accurately.

In a soil-structure interaction problem, the structure (near field) can be modelled using either the conventional finite element method or the scaled boundary finite element method. The soil (far field) is modelled by the scaled boundary finite element method. As shown in Figure 1, the relationship between the near field and the far field, in the time domain can be expressed in terms of the displacement vector \( \{u(t)\} \) and the interaction force vector \( \{R(t)\} \). The interaction force-displacement relationship is given in (Wolf and Song, 1997) as,

\[
\{R(t)\} = [C_\infty]\{\ddot{u}(t)\} + [K_\infty]\{u(t)\} + \int_0^t [S^{\infty}_r(\tau)]\{u(t - \tau)\}d\tau,
\]

where \([C_\infty]\) and \([K_\infty]\) are constant dashpot and spring matrices, respectively. The first two terms in Equation (1) represent the instantaneous displacement response, whereas the convolution term corresponds to the lingering response. \([S^{\infty}_r(t)]\) is the so-called displacement unit-impulse response matrix. An efficient algorithm for calculating \([S^{\infty}_r(t)]\) by using SBFEM has been proposed (Chen, et al., 2014), in which a piece-wise linear approximation of \([S^{\infty}_r(t)]\) with an extrapolation parameter and a truncation time is introduced to solve wave propagation problems in 2D half-space. The idea of using a piece-wise linear approximation has first been introduced in a conference paper by Radmanovic and Katz (2010).

![Figure 1. Soil-structure system. (a) Structure (near field); (b) Soil (far field)](image)

In this paper, the proposed method (Chen, et al., 2014) is further extended to solve soil-structure interaction problems in 3D half-space. Due to the large size of the computational domain in three dimensions, localization techniques are introduced. They are presented in the following order.

In Section 2, a recursive formulation is introduced to represent the interaction force-displacement relationship. The displacement unit-impulse response matrix is divided into a finite number of linear segments and truncated at a certain time, so that the interaction force at any time only depends on a finite number of the previous displacements, achieving localization in time. In Section 3, the whole unbounded domain is divided into several subdomains, so that the fully coupled displacement unit-impulse response matrix is decoupled, achieving localization in space. In Section 4, these localization methods are verified by a numerical example. Conclusions are presented in Section 5.

**LOCALIZATION IN TIME**

In Chen, et al., (2013), the displacement unit-impulse response matrix \([S^{\infty}_r(t)]\) is calculated by assuming a piece-wise linear variation, as shown in Figure 2. After a certain time \(t_N\), referred to as the
truncation time, the displacement unit-impulse response is assumed to be zero. By using the z-transform, the convolution term in Equation (1),

\[ \{Q(t)\} = \int_0^t [S^\infty_r(\tau)]\{u(t-\tau)\}d\tau, \]  

(2)

can be written in a recursive form as,

\[ \{Q\}_n = 2\{Q\}_{n-1} - \{Q\}_{n-2} + \sum_{i=0}^{N} [C]_i \{u\}_i, \]  

(3)

where \{Q\}_n is \{Q(t)\} at discrete time \( t = n\Delta t \). The detailed derivation of Equation (3) can be found in Chapter 5, Wolf, (1988).

LOCALIZATION IN SPACE

In the original method using SBFEM to model the unbounded domain (Wolf and Song, 1997), the whole unbounded domain was treated as one subdomain. This results in a fully populated displacement unit-impulse response matrix, whose size is equal to the total number of degrees of freedom on the interface. Alternatively, the unbounded domain can be divided into several subdomains, and thus the interaction force-displacement relationship (1) can be broken into several smaller matrix-vector operations.

Note that, dividing the unbounded domain into multiple subdomains is an approximation, i.e. spurious reflections may occur on the interface between unbounded subdomains. Also note that wave motion tends to become one-dimensional with increasing distance from the source, as stated in Chapter 3.9 in (Wolf, 1988). Thus, the spurious reflections on the interface can be reduced by placing the artificial boundary further away from the domain of interest. The domain within the artificial boundary is referred to as the near field, which is modelled by FEM or SBFEM and the domain outside the artificial boundary is referred to as the far field, which is modelled by SBFEM. By doing so, the computational effort for calculating the interaction force-displacement relationship is reduced. This is illustrated in the following parameter study.

NUMERICAL EXAMPLE

A cubic foundation embedded in homogeneous half-space is investigated, as shown in Figure 3. The foundation has a size of \( 2b \times 2b \times b \) and we let \( b = 1 \) m. The artificial boundary, in \( 2d \times 2d \times d \) shape, encloses the foundation. The surface centre of the near field coincides with the surface center of the foundation, which is denoted as \( O \) in Figure 3. The concrete foundation has a Young’s modulus of \( E_f = 15 \) GPa, Poisson’s ratio of \( \nu_f = 0.2 \) and mass density of \( \rho_f = 2500 \) kg/m\(^3\). The adjacent soil is described by a Young’s modulus of \( E = 312.5 \) MPa, Poisson’s ratio of \( \nu = 0.25 \) and mass density of \( \rho = 2000 \) kg/m\(^3\).
The near field is modelled with 8-node isoparametric block finite elements with uniform size of $0.1b \times 0.1b \times 0.1b$. Generally, for modelling dynamic problems using the finite element method, at least ten nodes per shear wave length are required. Therefore, the minimum shear wave length this mesh can model is $b$, which is used as a characteristic parameter to measure the distance between the artificial boundary and actual foundation-soil interface. On the near field / far field interface, 4-node isoparametric surface scaled boundary finite elements are used, which coincide with the FE mesh of the near field. The scaling centre is located at point $O$. First the unbounded domain is treated as one subdomain. Then, each element is treated as one subdomain, and a group of unit-impulse response matrices for every subdomain is calculated, marked as '1 element' subgroup. Then, every $2 \times 2$ and $3 \times 3$ elements are grouped as one subdomain, marked as '4 elements', and '9 elements' subgroups, respectively. These sizes of the subdomain can also be interpreted as $10\%$, $20\%$ and $30\%$ of the characteristic length $b$, which is the minimum wave length.

A vertical uniformly distributed load $p(t)$ is applied on the surface of the foundation, and its time history is shown in Figure 4. The vertical displacement of the surface centre of the foundation, point $O$, is evaluated. The displacement unit-impulse response matrix is calculated with a time step size $0.02b/c_p$ for five steps, resulting in a truncation time of $t_N = 0.1b/c_p$. In the time domain analysis, the time step size is $\Delta t = 0.01b/c_p$. For verification, an extended FE mesh is used, where a domain of size $20b \times 20b \times 20b$ is modelled using the finite element method as described above.

The results are presented in four groups, corresponding to different locations of the artificial boundary, i.e. $d=b$, $d=2b$, $d=3b$ and $d=4b$. In each group, the displacement is calculated using one subdomain, marked as 'coupled', and then either of the three different subdomain division schemes explained above. The results are shown in Figure 5.
As can be seen in Figure 5, for the case \( d=b \), i.e. when the artificial boundary coincides with the actual foundation-soil interface, using multiple unbounded subdomains may cause large errors. The error is reduced by placing the artificial boundary further away from the foundation. For the case \( d=2b \), the ‘9 elements’ scheme is already acceptable for engineering practice, whose relative error is less than 5%, and for the case \( d=3b \), all the three spatial decoupling schemes lead to only marginal errors (for ‘1 element’ scheme, the relative error is already less than 5%). The above schemes, together with the ‘1 element’ scheme for \( d=4b \), provide acceptable accuracy. The CPU time for calculating the interaction force-displacement relationship, Equation (1), using the different schemes is recorded and shown in Figure 6.

It can be seen that the spatial decoupling leads to a significant reduction of computational effort. However, as the artificial boundary is located further away from the domain of interest, the size of the near field is increased. This will lead to additional degrees of freedom in the near field and to an increase of the total computational size. In large scale engineering practice, the original near field may be very large and the increase of the additional near field due to further location of the artificial boundary can only be marginal.
CONCLUSION

In this paper, the numerical modelling of 3D dynamic soil-structure interaction problems using SBFEM is addressed. Due to the large geometric size in three dimensions, two essential localization techniques are introduced: (1) Temporally, the displacement unit-impulse response matrix for the unbounded domain calculated from the scaled boundary finite element method is divided into several linear segments. By using the z-transform, the time consuming convolution integral representing the force-displacement relationship on the interface is transformed into a recursive formulation, where only a small number of previous steps are involved. Together with the introduction of the truncation time, the total number of the operations regarding the convolution integral is reduced significantly; (2) Spatially, since the unbounded domain is divided into a number of independent subdomains, the fully coupled displacement unit-impulse response matrix for the unbounded domain can be broken into several small matrices, so that the computational size in space is reduced. As the interface between two subdomains will cause discontinuity, the artificial boundary needs to be placed a certain distance away from the domain of interest.

REFERENCES


Figure 6. CPU time required to evaluate the interaction force-displacement relationship