Estimating and measuring impact forces by projectiles

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ESTIMATING AND MEASURING IMPACT FORCES BY PROJECTILES

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ABSTRACT

The impact action of a projectile can cause the target element to deflect as if the action was a static force and can also inflict damage to the target in the vicinity of the point of contact. In extreme conditions, perforation or indentation into the target can occur because of the very high contact force generated by the impact. It should be noted that the amount of contact force occurring on impact, which normally lasts for only a few milliseconds, can be many times higher than the amount of equivalent static force required to produce similar amount of deflection. This paper aims at introducing analytical and experimental techniques for obtaining both types of forces, and contact force in particular (as is often not measured and difficult to estimate).

KEYWORDS

Impact action, contact force, quasi-static force, projectiles.

INTRODUCTION

In engineering practices, an impact action is often represented by an equivalent quasi-static force which is expected to generate a similar amount of deflection. As a roofing tile, or a glass pane, is subject to the impact action of a hailstone, or debris, object the amount of bending moment and deflection (and hence flexural tensile stresses) generated by the impact can be estimated by the analysis of the static action. This simple format of quantifying impact actions is convenient for structural design purposes. It should be noted that, the amount of contact force generated by the impact cannot be determined by considering the projectile alone given that it is also dependent on the interaction between the impactor and the target. This contact force is the cause of localised stresses generated by the impact surrounding the point of contact especially when subjected to high-speed impact. It is responsible for the risks of localised phenomena such as denting, local crushing or...
perforation. The important distinction between the contact force and quasi-static force in the context of impact actions is still not widely understood amongst practitioners and few guidelines have been developed for their estimations. Limitations of quasi-static force provisions have never been well explained. Consequently, many engineers are not confident in making estimates of impact actions unless specific stipulations are provided by the design codes of practices.

Engineers tend to rely on the use of proprietary software (e.g. LS-DYNA as “black boxes” for performing rigorous computational simulations) or on physical experimentations. Whilst maximum deflection of the target is always measured in an impact experiment the amount of maximum contact force generated by the impact surrounding the point of contact is often not measured (as is difficult to do so without interfering with the impact). What is usually recorded from a typical impact experiment is whether the specimen fails, or remains intact, in a designated impact scenario. The only other observations that can be made are the amount of damage that has been inflicted onto the specimen. Whist experimentations involving the use of the gas gun is common and test data is abundant the amount of potentially useful information that can be retrieved from those experiments to guide design and to quantify risk is very limited. Experimental data on contact force in particular is very scarce because of challenges with taking representative measurements. Forces are normally measured by load cells in physical experiments (e.g. Atas et al. 2011; Belingardi and Vadori 2002; Dhakal et al. 2012; Ghelli and Minak 2011; Zineddin et al. 2007). However, placing the load cell in front of the target for the direct measurement of the contact force can result in damage to the instrument following repetitive testings because of the abrasive nature of certain impact actions (Zhang and Sharf 2010). Any attempt to protect the load cell such as the use of a shield, or cushion, would only compromise the accuracies of the measurements if the actual impact to be modelled is without any protection. Placing the load cell behind the target would only result a reaction force to be observed instead of the contact force as the inertial resistance from the target itself would not be included in the measurement.

The objective of the paper is to introduce inexpensive methodologies for estimating and measuring contact forces. An introduction to the concept of contact force and quasi-static force is presented in section 2. A simple and conservative hand calculation method for estimating contact force is presented in section 3 using the impact of a cricket ball as example for illustration. The same example has been used in section 4 to provide a better match with results from more comprehensive 2DOF modelling. Results of contact forces obtained from these analytical techniques are compared with experimentally measured results. Custom designed tubular device, with or without gas gun fittings, for measuring impact forces are introduced in section 5.

CONTACT FORCE

Quasi-static forces generated by the impact can be exceeded significantly by the contact force because of contributions from the inertial resistance of the target (Figure 1). The two types of forces are best illustrated by a two-degree-of-freedom (2DOF) system model comprising two spring-connected lumped masses. The frontal lumped mass ($m_1$) represents the impactor whereas the second lumped mass ($m_2$) at the rear represents the targeted element. The deformation of the impactor as well as the indentation into the surface of the target is emulated by the compression of the frontal spring (with spring stiffness coefficients $k_1$ and $p$). The displacement response behaviour of the target structure is reflected in the stretching, and compression, of the rear spring (with spring stiffness $k_2$).

Consider an example (Figure 1) where a cricket ball weighting 0.156 kg and travelling at 6.0 m/s impacts on a 1.950 kg target lumped mass (Yang 2013). The target lumped mass is supported by a rear spring with stiffness 23.4 kN/m. A strain gauge linear displacement transducer (LDT) was used to take measurement of the target displacement ($x_2(t)$). The magnitude of the reaction force ($F_R$) can be taken as the product of the spring stiffness ($k_2$) and spring shortening, $x_2(t)$, whereas the magnitude of the inertia force ($F_I$) is taken as the product of mass, $m_2$, and acceleration, $\ddot{x}_2(t)$. From vertical equilibrium of forces (Figure 1), Eq. 1 was derived for determining the magnitude of the contact force ($F_c$).

ACMSM23 2014 1052
Figure 1. Cricket ball impact

\[ F_c = F_I + F_R = m_1 \ddot{x}_1(t) + k_2 x_2(t) \]  

(1)

Results of the contact force and reaction force time histories are depicted in Figure 2. The value of peak contact force (1710 N) was found to be more than ten times higher than the maximum reaction force of 145 N. The contact force only lasted for 1.76 ms whilst the reaction force lasted for 29.06 ms.

Figure 2. Contact force and reaction (quasi-static) force time histories

ENERGY BASED MODEL

The contact force – displacement \((F_c - \delta)\) relationship can be expressed in Eq. 2. This \(F_c - \delta\) behaviour is represented by the frontal spring in the 2DOF spring connected lumped mass system which is not to be confused with the behaviour of the rear spring in support of the target lumped mass. Assuming that the target is very stiff (e.g. \(k_2\) close to infinity) with extreme value of mass (e.g. \(m_2\) close to infinity). Should values of \(k_n\) and \(p\) be known the maximum value of the displacement \((\delta)\) can be estimated using Eq. 4 which is expressed in terms of basic impact parameters \((m_1\) and \(v_0)\):

\[ F_c = k_n \delta^p \quad \text{where} \quad \delta = x_1 - x_2 \]  

(2)

\[ \frac{1}{2} m_1 v_o^2 = \int_0^{\delta_{\text{max}}} F_c d\delta = \int_0^{\delta_{\text{max}}} k_n \delta^p d\delta = \frac{k_n}{p+1} \delta_{\text{max}}^{p+1} \]  

(3)

\[ \delta_{\text{max}} = \left( \frac{p+1}{2k_n} m_1 v_o^2 \right)^{\frac{1}{p+1}} \]  

(4)

Eqs. 3 and 4 are based on the assumption that no energy is dissipated in the loading phase of the impact (when the impactor is compressed). Mitigating effects which are derived from interactions between the impactor and the target have also been ignored. In spite of these assumptions reasonable results can be obtained from these equations in a typical scenario wherein the duration of contact is an order of magnitude shorter than the time taken by the target to displace. The value of \(F_c\) is accordingly defined by the following expression based on substituting Eq. 4 into Eq. 2:

\[ F_c = k_n \left( \frac{p+1}{2k_n} m_1 v_o^2 \right)^{\frac{1}{p+1}} \]  

(5)

Consider the aforementioned example of impact by the cricket ball. The parameters of the frontal spring stiffness \((k_n = 13.5 \text{ MN/m}^p)\) and the exponent \((p = 1.56)\) with the identical impactor were
obtained by Sun et al. (2014). Substitution those values into Eqs. 4 and 5 generates the following expressions:

\[ \delta_{\text{max}} = 8.74 \times 10^{-3} v_0^{0.78} \quad \text{and} \quad F_c = 230 v_0^{1.22} \]  

(6)

By the use of the stated relationships, the amount of contact force generated by the impact of a cricket ball at incident velocity of 6 m/s is equal to 2050 N. Comparison of this estimates with experimental results is shown in Figure 3. It is shown that the proposed model provides a conservative estimate due to the initial assumption of zero energy loss in the loading phase of the impact.

\[ F_c = D_n \delta^p \delta + k_n \delta^p \]  

(7)

By considering dynamic equilibrium of forces, the equations of motion for the non-linear visco-elastic 2DOF model can be written as Eq. 8. The numerical implementation scheme in solving for Eq. 8 can be achieved by time integration method, details are not provided.

\[
\begin{aligned}
D_n \ddot{\delta} + k_n \delta^p &= m_i \ddot{x}_i \\
D_n \ddot{\delta} + k_n \delta^p &= m_2 \ddot{x}_2 + k_2 x_2
\end{aligned}
\]  

(8)

With the same cricket ball impact, solutions for the time histories of the contact force and reaction force have been shown to match well with experimental observations (Figure 5). Comparisons of results are as presented in Table 1. It can be seen that the 2DOF model provides reasonable estimates of contact force with only 3.0 % discrepancy in comparison with that obtained from experiment. Contact force obtained from an energy based model provides a lot more conservative estimates.
Table 1. Maximum contact force comparison

<table>
<thead>
<tr>
<th>Maximum Contact Force</th>
<th>Discrepancy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental Measurement</td>
<td>1710 N</td>
</tr>
<tr>
<td>Energy Based Method</td>
<td>2050 N</td>
</tr>
<tr>
<td>2DOF Non-linear Visco-elastic Model</td>
<td>1761 N</td>
</tr>
</tbody>
</table>

EXPERIMENTAL DEVELOPMENT IN MEASURING CONTACT FORCE

Measurement of Contact Force by a Drop Tube Device

High contact force generated by an impactor can result in perforation or denting of the target. A custom made measuring device (Figure 6 (a)) has been built by the authors and co-worker for measuring the contact force generated by an impact. The fundamental operational principle of the measurement device is to incorporate a physical model of a spring connected lumped mass system within the device in order that the time history of the contact force can be inferred from the directly measured displacement time-histories of the lumped mass. From Eq. 1, the contact force can be calculated as the sum of the reaction force (inferred from the retraction of the rear spring) and the inertia force generated from within the targeted object (inferred from the accelerations of the object). The apparatus of Figure 6 has been tried out with tests on common spherical objects such as cricket balls which have been used as an example in previous sections. It is shown that much of the contact force was reacted by the inertia force of the target whereas contributions by the reaction force were very minor.

(a) Drop tube test (b) Tubular device enhanced by gas gun fitting

Figure 6. Experimental apparatus and setup

Measurement of Contact Force by Tubular Device

Experiments conducted to date using the drop tube device of Figure 6 (a) were of very modest velocity (up to 6 m/s). An enhanced version of the device featuring a gas gun fitting has been built to extend this methodology to high speed impact scenarios (Figure 6 (b)). The operational principle of the gas gun fitted device is the same as that of the drop tube device. Importantly, an incident velocity
up to 50 m/s can be achieved by the use of the gas gun. A velocity sensor is used to measure the incident velocity of the impactor immediately prior to the occurrence of impact. A laser sensor is used in place of LDT for measuring the target displacement time-histories and an accelerometer is used for measuring the acceleration of the dummy lumped mass for determination of the time-histories of the contact force.

CONCLUSIONS

The distinction between quasi-static force and contact force was first clarified emphatically in the early part of the paper. Contact force which can be an order of magnitude higher than the quasi-static force is responsible for localised failure such as punching and perforations. Two methodologies: 1) energy based model and 2) 2DOF non-linear visco-elastic model, have been introduced for obtaining estimates of the contact force using cricket ball as the example impactor. The energy based model is a simple way of predicting conservative contact force estimates. The 2DOF model is a more comprehensive method which takes into account energy dissipation. Values of the contact force that have estimated by the proposed methodologies have been verified by custom made physical devices. The analytical and experimental techniques introduced in this paper enable accurate predictions of impact actions for assessing the potential performance of fragile when subject to the impact actions of a projectile.

REFERENCES


