Calculating pseudo-steady-state horizontal oil well productivity in rectangular drainage areas using a simple method

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Calculating Pseudo-Steady-State Horizontal Oil Well Productivity in Rectangular Drainage Areas Using a Simple Method

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To determine the economical feasibility of drilling a horizontal well, engineers need reliable methods to estimate its productivity. In this work, a simple-to-use method is developed to rapidly estimate a pseudo-steady-state horizontal well's productivity. Estimations are found to be in excellent agreement with the reliable data in the literature, with average absolute deviation being less than 1%. The tool developed in this study can be of immense practical value for petroleum engineers to make a quick check on a pseudo-steady-state horizontal well's productivity at various conditions without opting for any field trials. The predictive tool is simple and straightforward, and it can be readily implemented in a standard spreadsheet program. The prime application of the method is as a quick-and-easy evaluation tool in conceptual development and scoping studies where horizontal wells are being considered. The method may also serve as a benchmark in numerical reservoir simulation studies.

Keywords Horizontal well; Predictive tool; Productivity; Rectangular drainage areas; Shape-related skin factors; Vandermonde matrix

Introduction

Transient pressure analysis of horizontal wells is more complex than that of vertical wells, because most horizontal well models assume that horizontal wells are perfectly horizontal and are parallel to the top and bottom boundaries of the reservoir (Chaudhry, 2004; Earlougher, 1977; Peaceman, 1990; Abdelgawad and Malekzadeh, 2001). When the fluid mass situated at the drainage boundary starts moving towards the producing well, the pseudo-steady state begins (Furui et al., 2003; Penmatcha and Aziz, 1999). This pseudo-steady state is also called a semi-steady state or a depletion state, and it shows the reservoir has reached a point where the pressure at all reservoir boundaries and also the average reservoir pressure will decrease over time as more and more fluid is withdrawn from the reservoir (Malekzadeh and Abdelgawad, 1999; Voronich et al., 2011).

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In this work, we present an analytical method to estimate the pseudo-steady-state productivity index of horizontal wells in a rectangular drainage area, with box-shaped reservoirs. Our prime interest is the productivity at long production times, that is, when the pressure distribution and flow pattern in the reservoir are dominated by the outer boundaries of the reservoir.

Clonts and Ramey (1986) derived the solution for the pressure transient response of horizontal wells by combining instantaneous source functions with the Newman product method. Recently, Hagoort (2009) developed an analytical method for the estimation of the productivity of an infinite-conductivity horizontal well in a closed, rectangular box-shaped reservoir. Daviau et al. (1985) used the product of the source functions for isotropic reservoirs with infinite, impermeable, or constant pressure outer boundaries. Ozkan et al. (1989) and Joshi (1991) presented a comprehensive solution for infinite, anisotropic reservoirs with either uniform influx or an infinite-conductivity horizontal well. Several authors (Daviau et al., 1985; Odeh and Babu, 1990; Lu and Tiab, 2008; Zeng et al., 2009) have applied the source function method to solve for the pressure in bounded reservoirs. Goode and Thambbynayam (1987) considered the horizontal well-bore as a strip source, with a uniform flux distribution along the length. They combined Laplace and finite Fourier cosine transforms to solve the pressure response in a horizontal well.

In general, drilled horizontal well-bores are rarely horizontal, with many variations in the vertical plane along the well length affecting the pressure gauge inserted at the producing end of a horizontal well; in addition, calculation is not straightforward because horizontal wells exhibit negative skin factors (Fetkovich and Vienot, 1985; Ehlig-Economides and Ramey, 1981; Wan et al., 2000). Also, it is difficult to estimate exact production length of a long horizontal well (Chaudhry, 2004).

Mutalik et al. (1988) have reported data for the shape factors and the corresponding equivalent skin factors, $s_{CA,h}$, for horizontal wells located at various positions within the drainage volume (Chaudhry, 2004). This method assumes a horizontal well drilled in a bounded reservoir as an infinite-conductivity well. The following equation can be used to calculate the productivity of a horizontal oil well (Chaudhry, 2004):

$$J_h = \frac{q_o}{P_R - P_{wf}} = \frac{k h}{141.2 \beta_o \mu_o} \left( \frac{1}{\ln \left( \frac{r_e}{r_w} \right) - A' + s_f + s_m + s_{CAh} - C' + Dq_o} \right)$$

where

$$r_e' = \sqrt{\frac{A}{\pi}}$$

$$s_f = -\ln \left( \frac{L}{4r_w} \right)$$

The estimation of productivity of a horizontal oil well depends on the pseudo-skin factor for centrally located wells within various drainage areas.

In view of the above mentioned issues, it is necessary to develop an accurate and simple correlation that is easier than existing approaches, less complicated, and with fewer computations for predicting the pseudo-skin factor as a function...
of dimensionless length \((L_D)\) and the ratio of horizontal well length over drainage area side \(\left(\frac{L}{2h}\right)\) for square and rectangular shapes with ratios of sides 1, 2, and 5.

\[
L_D = \frac{L}{2h} \sqrt{k_y} \sqrt{\frac{k_y}{k_h}}
\]

(4)

This article discusses the formulation of such a predictive tool in a systematic manner along with an example to show the simplicity of the model and usefulness of such tools. The proposed method is an exponential function, which leads to well-behaved (i.e., smooth and non-oscillatory) equations enabling more accurate and non-oscillatory predictions, and this is the distinct advantage of the proposed method (Bahadori, 2010, 2012; Bahadori and Nouri, 2012).

**Methodology for the Development of Novel Correlation**

The primary purpose of the present study is to accurately correlate the pseudo-skin factor as a function of dimensionless length \((L_D\) from Equation (4)) and the ratio of horizontal well length over drainage area side \(\left(\frac{L}{2h}\right)\) for square and rectangular shapes with ratios of sides 1, 2, and 5.

The Vandermonde matrix is a matrix with the terms of a geometric progression in each row, i.e., an \(m \times n\) matrix (Horn and Johnson, 1991).

\[
V_{ij} = \alpha_j^{-1}
\]

(6)

for all indices \(i\) and \(j\). The determinant of a square Vandermonde matrix (where \(m = n\)) can be expressed as (Horn and Johnson, 1991):

\[
\det(V) = \prod_{1 \leq i < j \leq n} (\alpha_j - \alpha_i)
\]

(7)

The Vandermonde matrix evaluates a polynomial at a set of points; formally, it transforms coefficients of a polynomial \(a_0 + a_1x + a_2x^2 + \ldots + a_{n-1}x^{n-1}\) to the values the polynomial takes at the point’s \(\alpha_i\). The non-vanishing of the Vandermonde determinant for distinct points \(\alpha_i\) shows that, for distinct points, the map from coefficients to values at those points is a one-to-one correspondence and thus that the polynomial interpolation problem is solvable with a unique solution; this result is called the unisolvence theorem (Fulton and Harris, 1991). They are thus useful in polynomial interpolation, since solving the system of linear equations \(Vu = y\) for \(u\) with \(V\) an \(m \times n\) Vandermonde matrix is equivalent to finding the coefficients \(u_j\) of the polynomial(s) (Fulton and Harris, 1991).

\[
P(x) = \sum_{j=0}^{n-1} u_j x^j
\]

(8)
For degree \( n \) which has (have) the property:

\[
P(x_i) = y_i \quad \text{for} \quad i = 1, \ldots, m
\]

(9)

The Vandermonde matrix can easily be inverted in terms of Lagrange basis polynomials: each column is the coefficients of the Lagrange basis polynomial, with terms in increasing order going down. The resulting solution to the interpolation problem is called the Lagrange polynomial (Horn and Johnson, 1991; Fulton and Harris, 1991); suppose that the interpolation polynomial is in the form:

\[
P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0
\]

(10)

The statement that \( p \) interpolates the data points means that

\[
p(x_i) = y_i \quad \text{for all} \quad i \in \{0, 1, \ldots, n\}.
\]

(11)

If we substitute Equation (6) in here, we get a system of linear equations in the coefficients \( a_k \). The system in matrix-vector form reads (Bair et al., 2006):

\[
\begin{bmatrix}
x_0^n & x_0^{n-1} & x_0^{n-2} & \cdots & x_0 & 1 \\
x_1^n & x_1^{n-1} & x_1^{n-2} & \cdots & x_1 & 1 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
x_n^n & x_n^{n-1} & x_n^{n-2} & \cdots & x_n & 1
\end{bmatrix}
\begin{bmatrix}
a_n \\
a_{n-1} \\
\vdots \\
a_0
\end{bmatrix}
=
\begin{bmatrix}
y_0 \\
y_1 \\
\vdots \\
y_n
\end{bmatrix}.
\]

(12)

We have to solve this system for \( a_k \) to construct the interpolant \( p(x) \). The matrix on the left is commonly referred to as a Vandermonde matrix (Bair et al., 2006).

**Development of Correlation**

The required data to develop this correlation includes the reported pseudo-skin factor data (Chaudhry, 2004; Mutalik et al., 1988) as a function of dimensionless length \((L_D\) in Equation (4)) and the ratio of horizontal well length over drainage area side \(\left(\frac{L}{2v_c}\right)\) for square and rectangular-shaped drainage areas with ratios of sides 1, 2, and 5. The following methodology has been applied to develop this correlation. First, pseudo-skin factor data are correlated as a function of dimensionless length \((L_D\) from Equation (4)) for several ratios of horizontal well length over drainage area side \(\left(\frac{L}{2v_c}\right)\), then, the calculated coefficients for these equations are correlated as a function of ratio of horizontal well length over drainage area side \(\left(\frac{L}{2v_c}\right)\). The derived equations are applied to calculate new coefficients for Equation (13) to predict pseudo-skin factor. Table I shows the tuned coefficients for Equations (14)–(17) for predicting pseudo-skin factor in horizontal wells. In brief, the following steps (Bahadori, 2010) are repeated to tune the correlation’s coefficients using MATLAB version 7.6.0.324 software (MathWorks, Natick, Mass.):

1. Correlate the shape-related skin factor or pseudo-skin factor data as a function of dimensionless length \((L_D\) from Equation (4)) for a given ratio of horizontal well length over drainage area side \(\left(\frac{L}{2v_c}\right)\).
2. Repeat step 1 for other ratios of horizontal well length over drainage area side \(( \frac{L}{2x_e} )\).

3. Correlate the corresponding polynomial coefficients that were obtained for different ratios of horizontal well length over drainage area side \(( \frac{L}{2x_e} )\) versus dimensionless length \((L_D \text{ from Equation (4)})\), \(a = f \left( \frac{L}{2x_e} \right)\), \(b = f \left( \frac{L}{2x_e} \right)\), \(c = f \left( \frac{L}{2x_e} \right)\), \(d = f \left( \frac{L}{2x_e} \right)\) (see Equations (14)–(17)).

Equation (13) represents the proposed governing equation in which four coefficients are used to correlate the shape-related skin factor or pseudo-skin factor data as a function of dimensionless length \((\frac{L}{2x_e})\) and the ratio of horizontal well length over drainage area side \((\frac{L}{2x_e})\), where the relevant coefficients have been reported in Table I for square and rectangular drainage areas.

\[
\ln(s_{CAh}) = a + \frac{b}{L_D} + \frac{c}{(L_D)^2} + \frac{d}{(L_D)^3}
\]  
(13)

where

\[
a = A_1 + B_1 \left( \frac{L}{2x_e} \right) + C_1 \left( \frac{L}{2x_e} \right)^2 + D_1 \left( \frac{L}{2x_e} \right)^3 
\]  
(14)
b = A_2 + B_2 \left( \frac{L}{2x_e} \right) + C_2 \left( \frac{L}{2x_e} \right)^2 + D_2 \left( \frac{L}{2x_e} \right)^3 \quad (15)

c = A_3 + B_3 \left( \frac{L}{2x_e} \right) + C_3 \left( \frac{L}{2x_e} \right)^2 + D_3 \left( \frac{L}{2x_e} \right)^3 \quad (16)

d = A_4 + B_4 \left( \frac{L}{2x_e} \right) + C_4 \left( \frac{L}{2x_e} \right)^2 + D_4 \left( \frac{L}{2x_e} \right)^3 \quad (17)

These optimum tuned coefficients help to cover \( L_D \) up to 100 and the ratio of horizontal well length over drainage area side \( \frac{L}{2x_e} \) up to 1. The optimum tuned coefficients given in Table I can be further retuned quickly according to the proposed approach if more data become available in the future.

In this work, our efforts directed at formulating a correlation can be expected to assist engineers for rapid calculation of shape-related skin factor or pseudo-skin factor as a function of dimensionless length \( L_D \) in Equation (4) and ratio of horizontal well length over drainage area side \( \frac{L}{2x_e} \) using an exponential function. The proposed novel tool developed in the present work is a simple and unique expression, nonexistent in the literature. Furthermore, the selected exponential function to develop the tool leads to well-behaved (i.e., smooth and non-oscillatory) equations enabling reliable and more accurate predictions.

Results

Figure 1 shows the schematic of a horizontal well located in a rectangular drainage volume. Figure 2 shows a screen page of the MATLAB-based computer program to calculate the shape-related skin factor or pseudo-skin factor in horizontal wells. Figures 3 and 5 show the results from the proposed method and excellent performance in the prediction of shape-related skin factor or pseudo-skin factor as a function of dimensionless length \( L_D \) from Equation (4) and the ratio of horizontal well length over drainage area side \( \frac{L}{2x_e} \) for various drainage areas. Figures 6 and 7 illustrate the excellent performance of the developed predictive tool for estimating

Figure 1. Schematic of a horizontal well located in a rectangular drainage volume.
shape-related skin factor or pseudo-skin factor in horizontal wells with square drainage area. Table II illustrates the accuracy of the proposed correlation for predicting the shape-related skin factor or pseudo-skin factor in comparison with some reported data (Chaudhry, 2004; Mutalik et al., 1988). The accuracy of the correlation in terms of average absolute deviation is less than 1%.

Figure 2. MATLAB-based software. (Figure provided in color online.)

Figure 3. Shape-related skin factor, $s_{CAH}$, for a horizontal well in a square drainage area $(xe/ye = 1)$. (Figure provided in color online.)
It is expected that our efforts in formulating a simple tool in this investigation will pave the way for arriving at an accurate prediction of shape-related skin factor or pseudo-skin factor that can be used by petroleum engineers for monitoring the key parameters periodically. A typical example is given below to illustrate the simplicity associated with the use of the proposed correlation for rapid estimation of shape-related skin factor or pseudo-skin factor. The tool developed in this study can be of immense practical value for petroleum reservoir experts and production

![Graph showing pseudoskin factor vs. dimensionless length](image)

**Figure 4.** Shape-related skin factor, $s_{CAh}$, for a horizontal well located in a rectangular drainage area ($xe/ye = 2$). (Figure provided in color online.)

![Graph showing pseudoskin factor vs. dimensionless length](image)

**Figure 5.** Shape-related skin factor, $s_{CAh}$, for a horizontal well located in a rectangular drainage area ($xe/ye = 5$). (Figure provided in color online.)
engineers to make a quick check of shape-related skin factor or pseudo-skin factor at various drainage areas without opting for any field trials. In particular, petroleum engineers would find the approach to be user-friendly, with transparent calculations involving no complex expressions.

Figure 6. The performance of predictive tool for predicting shape-related skin factor, $s_{C_{Ah}}$, for a horizontal well in a square drainage area ($xe/ye = 1$). (Figure provided in color online.)

Figure 7. The performance of predictive tool for predicting shape-related skin factor, $s_{C_{Ah}}$, for a horizontal well in a square drainage area ($xe/ye = 1$) from another viewpoint. (Figure provided in color online.)
Table II. Comparison of calculated shape-related skin factor with typical data

<table>
<thead>
<tr>
<th>Dimensionless length, ( \frac{L}{2\nu} )</th>
<th>Reported shape-related skin factor</th>
<th>Calculated shape-related skin factor using new correlation</th>
<th>Absolute deviation percentage</th>
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Average absolute deviation, percent: 0.85%
Example

A horizontal oil well, 2000 ft long, is drilled in a reservoir with the following characteristics: \( h = 100 \) ft, \( r_w = 0.39 \) ft, \( \phi = 4.9\% \), \( \beta_o = 1.215 \) rb/stb, \( \mu_o = 0.45 \) cP, \( \text{sm} = 0 \), \( D = 0 \), \( z_w = 30 \) ft, \( k_h = k_x = k_y = 1 \) mD, \( k_v/k_h = 0.5 \), \( A' = 0.738 \). Assuming square drainage area = 160 acres, one can calculate the pseudo-steady state horizontal well productivity as follows.

Solution:

\[
 r'_e = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{160 \times 43560}{\pi}} = 1489 \text{ ft}
\]

\[
s_f = -\ln \left( \frac{L}{4r_w} \right) = -\ln \left( \frac{2000}{4 \times 0.39} \right) = -7.1562
\]

Calculation of \( s_{CAh} \) using the new predictive tool:

\[
 L_D = \frac{L}{2h} \sqrt{\frac{k_v}{k_h}} = \frac{2000}{2 \times 100} \sqrt{1} = 10
\]

For a square drainage area shape \( 2x_c = 2y_c \). Therefore, \( 2x_c = \sqrt{160 \times 43560} = 2640 \) ft.

\[
 \frac{L}{2x_c} = \frac{2000}{2640} = 0.757
\]

Now, using the newly developed predictive tool \( s_{CAh} \) is calculated:

\[
a = 4.63957246 \times 10^{-1} \text{ (from Equation (14))}
b = 1.35865899 \text{ (from Equation (15))}
c = 2.20326526 \times 10^{-1} \text{ (from Equation (16))}
d = -4.90037342 \times 10^{-1} \text{ (from Equation (17))}
s_{CA,h} = 1.82492040 \text{ (from Equation (17))}
\]

The horizontal well productivity will be:

\[
 J_h = \frac{q_o}{P_R - P_{wf}} = \frac{kh}{141.2\beta_o\mu_o} \left( \frac{1}{\ln \left( \frac{L}{r_e} \right) - A' + s_f + s_m + s_{CA,h} - C' + Dq_o} \right)
\]

\[
 J_h = \frac{1 \times 100}{141.2 \times 0.45 \times 1.215} \left( \frac{1}{\ln \left( \frac{1489}{0.39} \right) - 0.738 - 7.1562 + 1.8249 + s_{CA,h} - 1.386 + 0} \right)
\]

\[
 = 1.644 \text{ Stb/(day.Psi)}
\]

It has very close agreement with the data reported by Chaudhry (2004), with less than 1% deviation.
Conclusions

In this work, simple-to-use equations, which are easier than existing approaches, less complicated with fewer computations, and suitable for petroleum engineers, are presented here for the estimation of shape-related skin factor or pseudo-skin factor as a function of dimensionless length ($L_D$ in Equation (4)) and ratio of horizontal well length over drainage area side ($\frac{L}{C_{17}}$) for various drainage areas. Unlike complex mathematical approaches for estimating shape-related skin factor or pseudo-skin factor, the proposed correlation is simple-to-use and would be of immense help for petroleum engineers, especially those dealing with reservoir engineering and production operations. Additionally, an engineer or practitioner can easily handle the level of mathematical formulations associated with the estimation of shape-related skin factor or pseudo-skin factor without any in-depth mathematical abilities. The example has clearly shown the simplicity and usefulness of the proposed tool. Furthermore, estimations are quite accurate as evidenced from the comparisons with literature data (with average absolute deviation being less than 1%) and would help in attempting engineering and operations in petroleum upstream industry with less time. The proposed method has a clear numerical background, wherein the relevant coefficients can be retuned quickly if more data become available in the future.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>tuned coefficient</td>
</tr>
<tr>
<td>B</td>
<td>tuned coefficient</td>
</tr>
<tr>
<td>C</td>
<td>tuned coefficient</td>
</tr>
<tr>
<td>C'</td>
<td>shape factor conversion constant</td>
</tr>
<tr>
<td>D</td>
<td>tuned coefficient</td>
</tr>
<tr>
<td>h</td>
<td>formation thickness, ft</td>
</tr>
<tr>
<td>i</td>
<td>index</td>
</tr>
<tr>
<td>j</td>
<td>index</td>
</tr>
<tr>
<td>$J_h$</td>
<td>horizontal well productivity index, bbl/(psi.day)</td>
</tr>
<tr>
<td>$K_h$</td>
<td>horizontal permeability, md</td>
</tr>
<tr>
<td>$K_v$</td>
<td>vertical permeability, Md</td>
</tr>
<tr>
<td>$m$</td>
<td>matrix row index for $m \times n$ matrix</td>
</tr>
<tr>
<td>$n$</td>
<td>matrix column index for $m \times n$ matrix</td>
</tr>
<tr>
<td>P</td>
<td>polynomial</td>
</tr>
<tr>
<td>$r'_e$</td>
<td>drainage radius, ft</td>
</tr>
<tr>
<td>$r_w$</td>
<td>well-bore radius, ft</td>
</tr>
<tr>
<td>$s_{CA,h}$</td>
<td>shape-related skin factor or pseudo-skin factor</td>
</tr>
<tr>
<td>$s_f$</td>
<td>negative skin factor of an infinite-conductivity, fully penetrating fracture of length (L)</td>
</tr>
<tr>
<td>$s_m$</td>
<td>mechanical skin factor, dimensionless</td>
</tr>
<tr>
<td>u</td>
<td>coefficient of polynomial</td>
</tr>
<tr>
<td>V</td>
<td>Vandermonde matrix</td>
</tr>
<tr>
<td>x</td>
<td>data point</td>
</tr>
<tr>
<td>y</td>
<td>data point</td>
</tr>
</tbody>
</table>
Greek Letters

$\alpha$ : matrix element
$\beta_o$ : oil formation volume factor, rb/stb
$\mu_o$ : porosity, percent
$\phi$ : oil viscosity, cP

References


