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MECHANICS BASED MODELS FOR DUCTILITY OF FIBRE REINFORCED CONCRETE BEAMS

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ABSTRACT

Steel fibre reinforced concrete (SFRC) is increasing in popularity as a material for structural design, due to an improved resistance to cracking and crack propagation, resulting in increased strength and toughness in flexure. However, there is a limited scope of application of fibre reinforced concrete (FRC) fuelled by a lack of understanding in FRC design for structural members. This research investigates the affect that adding steel fibres to a concrete matrix has on the moment-rotation ($M-R$) and load-deflection ($P_m-\delta_m$) responses of reinforced concrete (RC) beams. This includes the development of a numerical partial interaction (PI) reinforcement bar (rebar) load-slip ($P-\Delta$) model, numerical $M-R$ models and the determination of a $P_m-\delta_m$ relationship using an analytical method. The PI model, which can incorporate any generic bond stress-slip ($\tau-\delta$) relationship, simulates the bond and cracking behaviour of rebars embedded in SFRC. This allows the determination of the crack spacing, crack width and the longitudinal rebar $P-\Delta$ relationship. The $P-\Delta$ relationship is required to determine the $M-R$ response up until failure. It was found that the inclusion of fibres into the concrete matrix results in more, closely spaced cracks with reduced crack widths, due to the fibres restraining crack growth.

KEYWORDS

Fibre reinforced concrete beams, deflection, bond-slip, ductility, moment-rotation.

INTRODUCTION

The behaviour of FRC is well understood prior to crack initiation, as the mechanisms are similar to those in conventional RC. Once cracking has commenced however, the load is transferred to the fibres which provide additional tensile capacity to that already provided by the rebar. This additional capacity results in an improved strength and toughness of the member. Although it is well understood that the addition of fibres helps to improve the crack control of the member, uncertainties in modelling the flexural behaviour exist. The uncertainties are due to a lack of ability to accurately model the physical processes of the pull-out forces of the fibres, the rebar bond relationship of FRC and the resulting cracking behaviour. Consequently, the majority of studies undertaken on FRC have been experimental, with the aim of determining a $P_m-\delta_m$ relationship for a flexural member. This paper presents a numerical study into the effects of the addition of fibres into RC beams, in particular the $M-R$ and $P_m-\delta_m$ response of SFRC members. The specific objectives of the research are: (i) to develop a PI pull-pull model to simulate a flexural hinge in order to determine the $P-\Delta$ response and cracking behaviour of FRC and (ii) to develop two $M-R$ models. Whereas model 1 is based on a curvature approach with the assumption of a linear strain profile, model 2 is a rotation approach, based on a



fixed crack height assumption. The third objective of the study is to determine the P_m - δ_m response of a member using the M - R models and knowledge of cracking behaviour.

NUMERICAL PI PULL-OUT MODEL

Haskett et al.(2008) developed a numerical pull-out test model for RC which divides the pull-out test into a sequence of thin segments. Using extremely narrow segments allows the assumption that the slip is constant over each segment. After specifying the material and geometric parameters and assuming a slip at the loaded end, an iterative process can be used. A brief overview of this is given in Figure 1. Most importantly, for the development of a M - R model for FRC, Haskett’s model allows for the determination of the global P - Δ response from any local τ - δ relationship. The P - Δ response is required for the M - R model to determine the force carried by the rebar for a given rotation. The model required modification to account for fibres, have the ability to predict cracking, and to accurately replicate a flexural hinge. The model was also developed to incorporate a tri-linear stress- strain relationship for the rebar. As a result of these modifications, a FRC pull-pull test was developed as shown by Figure 2.

Development of Pull-pull Test for RC

The PI numerical pull-pull test differs from a normal pull-out test because the load, P_r is subjected to both ends of the reinforcement bar, rather than just one, as shown by Figure 2. It results in tension stiffening of the member. More importantly, however, the model allows for the calculation of the crack spacing, crack widths and corresponding reinforcement bar loads, which are required in the determination of the P_m - δ_m response. The pull-pull model was developed with the aid of the works of Muhamad et al. (2012) on tension stiffening in RC.

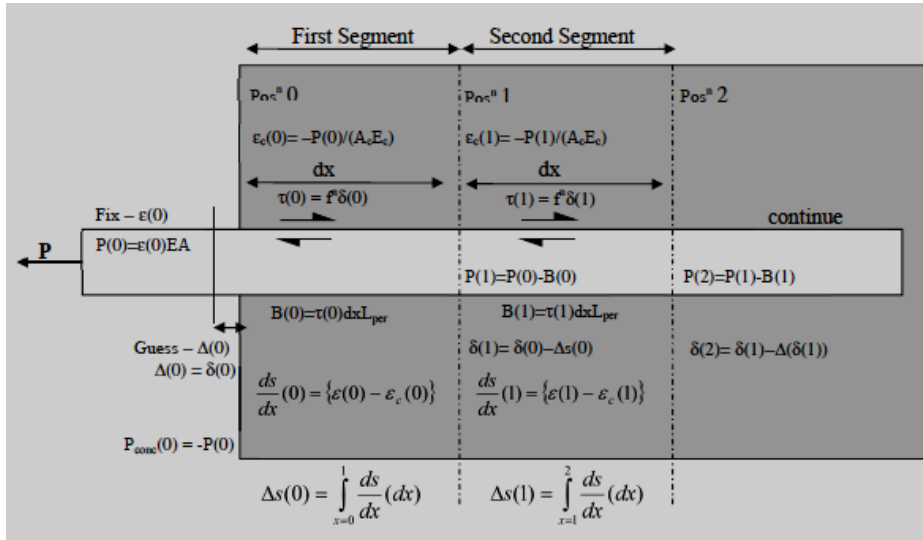


Figure 1. Graphical representation of the numerical pull-out test model

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$$P_c(i) + P_r(i) = \sigma_c A_c(i) + \sigma_r A_r(i) = 0 \tag{1}$$

$$\sigma_{st}A_{st}(i) + P_c(i) = \sigma_r A_r(i) = P_r(i) \quad (2)$$

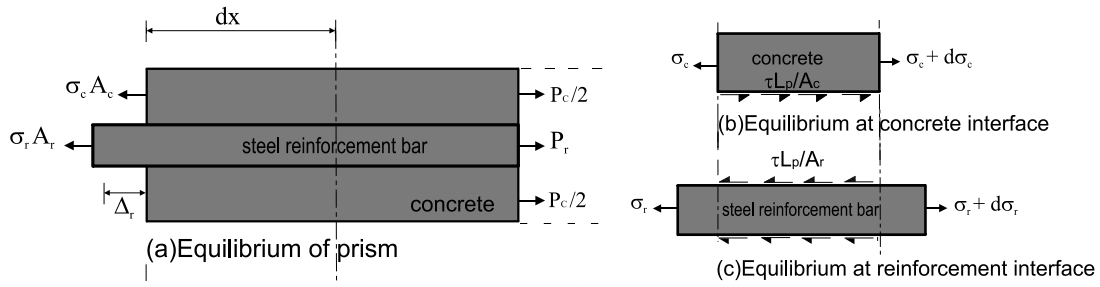


Figure 2. PI numerical pull-pull test

τ - δ Relationship for FRC

It has been established that the FRC τ - δ relationship differs from RC (Harajli et al. 2010; Jones et al. 2007). The FRC τ - δ relationship that best represents the fibre concrete-reinforcement bar interface relationship up until beam failure has been developed based on Harajli's et al.(2010) τ - δ relationship.

Parametric Study of P - Δ and τ - δ Responses

By applying the FRC τ - δ relationship described above into the PI numerical pull-pull model, P - Δ responses for FRC were developed. It was found that the addition of fibres provides additional tensile strength to that already provided by the rebar. From FRC τ - δ and P - Δ parametric studies undertaken, it was identified that a rebar still provides the majority of tensile strength in a cracked section. However, the fibres increase generally the strength of a beam and also the deflection and rotation capacity as discussed in subsequent sections. It was also found that the greater the fibre length, the greater the bond strength. Increasing the fibre diameter however, results in a larger sectional area over which the stress is distributed. This causes the bond stress for a given slip to decrease for an increased fibre diameter.

Predicting Cracking Behaviour in RC

In order to predict the cracking behaviour, an additional boundary condition was incorporated into the model. The boundary condition, $\sigma_c = f_{ct}$, specifies that cracking occurs when the stress in the concrete is equal to its tensile stress capacity, f_{ct} . This boundary condition must coincide with the original boundary conditions of Haskett's pull-out model, which occur when the relative slip of the rebar, δ and the slip strain, ds/dx converge to zero as given by $\delta = ds/dx = 0$. This provided the ability to predict the spacing along the beam of the first crack to form the 'primary' crack. The corresponding P - Δ provided the reinforcement bar capacity for a given crack width at this point. The subsequent crack spacing values for secondary and higher order cracks can then be determined. It was assumed from RC theory that secondary cracking occurs at the midpoint between primary cracks, tertiary cracking occurs at the midpoint between secondary cracks, and so on for higher order cracking. This is shown in Figure 3, where s_{pri} and s_{sec} represent primary and secondary crack spacing respectively.

Modification of Pull-pull Test for FRC

The same boundary condition as given by $\delta = ds/dx = 0$ can be used to determine crack spacing as what was used for RC. The fundamental difference between FRC and RC is in its ability to transfer stress through the fibres across the cracked section (Fantilli et al. 2009; Jones et al. 2007). Hence, the numerical PI pull-pull model was modified accordingly. This was done by incorporating a starting point for the tensile stress in the SFRC, which is equal to the fibre stress acting across the crack. The difference can be seen between Figure 4, which shows the PI pull-pull model convergence of boundary conditions for FRC.

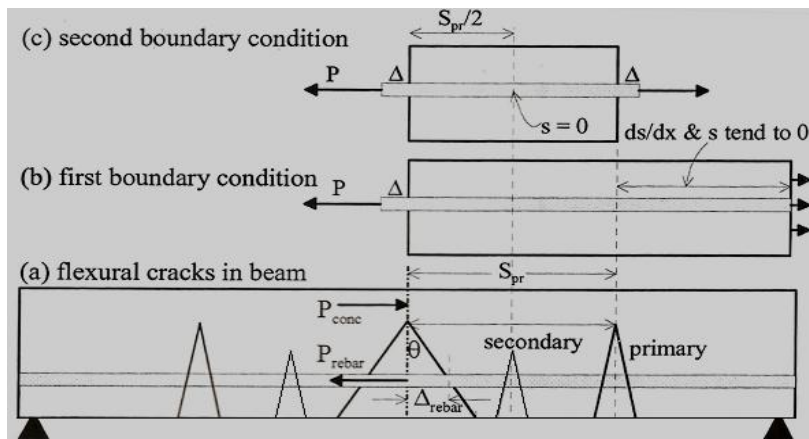


Figure 3. Formation of Primary and Secondary cracks in RC beam

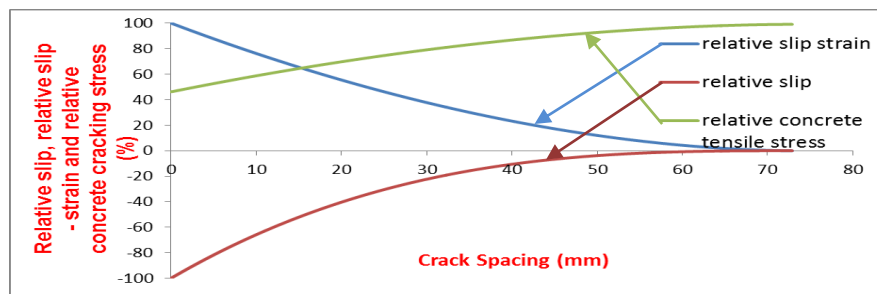


Figure 4. PI model convergence for FRC

Here, the relative concrete tensile stress for FRC begins at over 40% of the tensile stress of the concrete, whereas it begins at zero for RC. The stress capacities of the fibres at the initiation of cracking were determined using the tri-linear fictitious crack model developed by Kützing (2000). The σ_t parameter of Kützing's model was used as the residual stress capacity of the cracked concrete because it represents the stress at which load is first transferred to the fibres, or the stress at first cracking. Since FRC behaves in the same way as RC prior to cracking (Fantilli et al. 2009; Jones et al. 2007), it was assumed that the secondary crack would occur at half the primary crack spacing length for FRC, as assumed in the cracking of RC. Tertiary cracking is then assumed to again occur at the midpoint between the secondary cracks, and so on for higher order cracking. Analytical results indicate that with the addition of 2.5% fibre, the crack spacing is reduced by around 30%. This is consistent with research by Schumacher and Walraven (2009) and is due to the residual stress provided by the fibre across the concrete crack providing a starting point for the stress, meaning that less stress is required for the concrete to recuperate its tensional capacity. Since the rate at which stress transfers from the rebar in the concrete varies only minimally for FRC, this causes a reduction in crack spacing compared to RC. The secondary effect is that there is a substantial reduction in crack width, of over 60% in this instance for primary cracking. This is due to the fact that the slip is shared across a greater number of cracks than for RC. Crack width and spacing is also significantly reduced for higher order cracking. Finally, the addition of FRC delays the onset of cracking beyond primary cracking, which supports the notion of increased stiffness and flexural toughness for FRC beams.

M-R MODELLING FOR FRC

The idealised rigid body under analysis applied in this research is shown in Figure 5. It is an adaption of that used in RC analysis. Three material constitutive relationships were identified as having an influence on the $M-R$ of FRC that differ from RC. They are the compressive stress-strain relationship, the fibre-stress relationship and the $P-\Delta$ relationship of the rebar. The addition of fibres to the concrete matrix increases the ductility of the compressive stress-strain relationship and the theoretical model developed by Barros and Figueiras (1999) incorporates the required increased ductility in compression and was used in this research due to its accuracy across a range of fibre concentrations. The addition of

fibres to the concrete matrix increases the post-cracking tensile strength of the member. The stress that the fibres can transmit across the crack is proportional to the crack width (Kützing 2000)

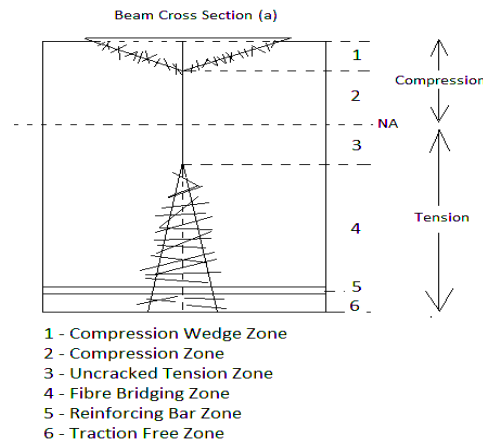


Figure 5. Idealised rigid body of FRC

Model Formulation and Validation

Model 1 extends a conventional RC *M-R* approach (Haskett et al. 2009) to include fibres, with the assumption of a linear strain profile. Model 2 extends the same *M-R* approach with the assumption of a fixed crack height. The two developed models were validated against experimental results to test their accuracy in simulating the *M-R* response until failure of the beam section. The experimental description of the FRC section is outlined in Figure 6 and the numerical *M-R* responses developed are compared to an experimental response in Figure 7. It can be observed that at ultimate limit state (ULS) the models have good agreement of both moment and rotation with the experimental data. At serviceability limit state (SLS), model 2 shows a good agreement with the experimental data. Model 1, while predicting the rotation accurately, overestimates the moment by approximately 15%. This is due to the large reduction in neutral axis depth of model 1, which is due to the linear- strain assumption. This increases the crack height, which for small rotations greatly increases the fibre tensile load and hence the moment at this point. Hence, at low rotations model 2 provides a more conservative estimation of moment capacity. Both the models fail at approximately 50% of the rotation of the experimental data. This is potentially due to an over-prediction of the models ultimate bond strength between the rebar and concrete, as this data is absent from Schumacher and Walravens' (2009) description.

CONCLUSIONS

Results from the PI pull-pull model illustrate a reduction in the crack spacing and crack widths for SFRC in comparison to RC and also show strong accuracy to experimental results. RC results from the model were also compared to closed form RC solutions from Muhammad et.al (2012) and showed strong accuracy. The two numerical *M-R* models (with fixed and varying crack heights) were able to simulate the *M-R* response for a range of fibre contents, fibre lengths, fibre diameters and concrete strengths. They demonstrated a close relationship to one another throughout the parametric study. The two models also showed a strong correlation to experimental *M-R* responses, accurately predicting ULS moments and rotations, with a reasonable correlation at SLS. Results show that additional fibres restrain the growth of all cracks, resulting in smaller crack widths with less rotation. In addition, the ability of the fibres to transfer stress across the cracked tensile region results in slight increases in the ultimate moment capacities and a more significant increase at SLS with increasing fibre concentrations.

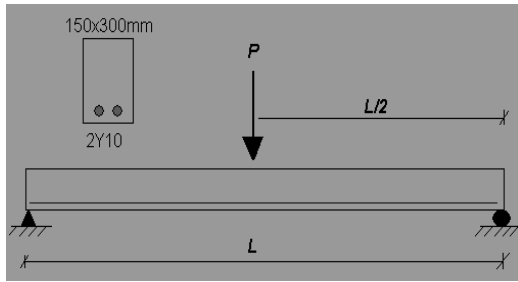


Figure 6. Beam loading arrangement

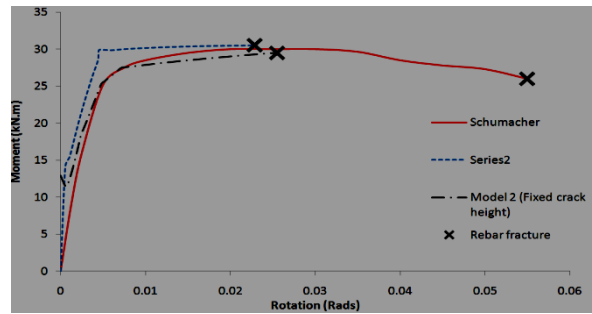


Figure 7. Experimental comparison of $M-R$ models

The $P_m-\delta_m$ responses calculated produced accurate deflections compared to experimental data at SLS, ULS and failure. As the $P_m-\delta_m$ curve generated is based on the simulated $M-R$ response, higher fibre concentrations will result in smaller deflections of beams at both SLS and ULS. Overall, the findings from this research demonstrate that there are significant advantages to including steel fibres into the concrete matrix. The addition of fibres results in many more smaller cracks; reducing the rotation and deflection of beams at both SLS and ULS.

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REFERENCES

- Barros, J.A.O., and Figueias, J.A. (1999). "Flexural behaviour of SFRC: Testing and Modelling", *Journal of Materials in Civil Engineering*, Vol.11, No.4, pp. 331-339.
- Fantilli, A.P., Mihashi,H., and Vallini, P., (2009). "Multiple cracking and strain hardening in fibre reinforced concrete under uniaxial tension", *Cement and Concrete Research*, Vol.39, No.12, pp.1217-1229.
- Jones, P.A., Austin, S.A., and Robins, P.J. (2007). "Predicting the flexural load-deflection response of Steel fibre reinforced concrete from strain, crack width, fibre pull-out and distribution data" *Material and Structures*, Vol.41, No.3, pp.449-463.
- Harajli M. H. (2010). "Bond Behavior in Steel Fiber-Reinforced Concrete Zones under Static and Cyclic Loading: Experimental Evaluations and Analytical Modeling" *Journal of Materials in Civil Engineering*, Vol.22, No.7, pp674-686.
- Haskett, M., Oehlers, D. J., and Mohamed Ali, M. S. (2008). "Local and global bond characteristics of steel reinforcing bars" *Engineering Structures*, Vol.30, No.2, pp.376-383.
- Haskett, M., Oehlers, D., Mohamed Ali, M.S, and Wu, C.(2009). "Rigid body moment-rotation mechanism for reinforced concrete beam hinges", *Engineering Structures* Vol.31, pp.1032-1041.
- Kützing, L. (2000). "Tragfähigkeitsermittlung stahlfaserverstärkter Betone", *B.G. Teubner*. Stuttgart Leipzig Wiesbaden.
- Muhamad, R., Mohamed Ali, M.S., Oehlers, DJ., and Griffith, J. (2012). The tension stiffening mechanism in reinforced concrete prisms. *Advances in Structural Engineering* Vol.15, No.12, pp.2053-2069 .
- Schumacher, P., and Walraven, J.C. (2009). Rotation Capacity of Self-Compacting Steel Fiber Reinforced Concrete, *Heron*, Vol. 54, No.2-3, pp.125-160.