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DESIGN OF REINFORCED CONCRETE CORBELS USING AS3600-2009

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ABSTRACT

First engineered in the 1900's (Hughes and Crisp, 2008), corbels have proved to be ideal for supporting significant loads from beams or slabs. With the present increasing demand for high strength concrete by designers and contractors, limitations in design formulas of current concrete structures codes become apparent and often iterative design processes are required. As such this study summarises design formula constraints from a number of design codes (AS3600-2009, AS3600-2001 and ACI318-11) followed by the use and verification of a design chart for concrete ranging from 20-100MPa. The chart is validated by comparison with published test results from previous research.

KEYWORDS

Corbel, strut-and-tie model, design chart.

INTRODUCTION

Background

Corbels can exhibit a range of failure mechanisms *viz* flexural tension failure due to the yielding of flexural reinforcement leading to crushing of concrete (Ozden and Atalay, 2011); shear compression failure from diagonal splitting along the compression strut (ACI318-11, 2011, Ozden and Atalay, 2011); sliding shear failure resulting in separation from the column or wall face (ACI318-11, 2011, Ozden and Atalay, 2011); splitting failure due to the vertical load being applied too close to the free end of the cantilever (Kriz and Raths, 1965); anchorage failures (Ahmad and Shah, 2009, Yousif, 2009); and, bearing failure causing cracking of concrete directly underneath, due to the bearing pad being too flexible or insufficient size (ACI318-11, 2011, Lu et al., 2009).

In accordance with Section 12.1.2 of the Australian Concrete Structures Code, AS3600-2009, designing a corbel for strength involves utilising one or a combination of the following methods; linear elastic (Section 6.4) or non-linear (Section 6.6) stress analysis, or strut and tie analysis (Section 7.5). It is generally assumed that the strut and tie method is a simple and effective method of analysing reinforced and prestressed concrete. As such the strut and tie method is recommended in the American ACI 318-11 and Australian AS3600-2009 Concrete Design Codes. Illustrated in Figure 1 is a strut and tie model for a corbel. This model idealises the complex flow of stresses in the corbel as axial elements of a truss (i.e. tensile, *T*, and compressive, *C*, elements in Figure 1). The concrete struts and reinforcing steel ties resist compressive and tensile stress fields, respectively. The locations of intersection of the struts and ties are termed nodes (see Figure 1) and they consist of hydrostatic stress fields in which both principal stresses are equal to the calculated compressive strength of the concrete strut. Overall, the strut and tie method aids the identification of load transfer characteristics in order to design reinforcement and determine the load carrying capacity of the structure.

Determining the forces in the compression strut and tension tie requires knowledge of either the inclination of the concrete strut or the depth of the compression region. Note that the concrete compressive strength and an efficiency factor (or reduction factor) is usually specified for the limiting compressive stress of the struts. The forces in the compression strut and tension tie can be determined by assuming trial values and carrying out an iterative procedure until all design criteria are satisfied and the area of tension reinforcement found.

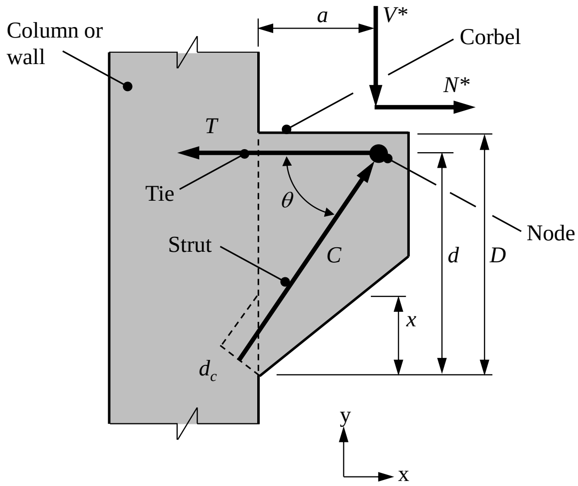

Figure 1. Corbel strut and tie model

High strength concrete corbels are becoming a frequent attribute in the building construction industry (Khalifa, 2012), however most code previsions catering only for normal strength concretes with strengths below 40MPa (Yong and Balaguru, 1994a). As such in this research a new design chart will be developed to aid design when using 20 to 100MPa concrete. Firstly, relevant design constraints from various major design codes (AS3600-2001, AS3600-2009 and ACI318-11) are highlighted and compared. The strut and tie method detailed in the AS3600-2009 concrete structures code, encompassing a trial and error procedure is then used to determine the amount of tension reinforcement required in a corbel to resist the anticipated loading. The design chart is then verified through comparison with a number of experimental works. Overall, the newly devised chart improves the design process by eliminating the need for an iterative design procedure.

Code Provisions for Corbel Design

The strength reduction factor (or capacity reduction factor) is applied when determining the design strength of the strut. Whereby, this factor reduces the useable concrete strength to take into account cracking of the struts and tensile strains transverse to the struts. AS3600-2001 (Clause 12.1.2.6) for the design of non-flexural members using the strut and tie method AS3600-2001 adopts a strength reduction factor of $\phi = 0.7$ irrespective of the mode of failure. ACI 318-11, Clause 11.8.3.1 states that corbel behaviour is predominantly controlled by shear, therefore, a single value of $\phi = 0.75$ is required for all design conditions.

Foster and Gilbert (1994) argued that this approach is inconsistent and it was therefore suggested that for corbel compression failure a value of $\phi = 0.6$ be used and for tension failure a value of $\phi = 0.8$ be adopted. AS3600-2009 (2009) now implements these values and defines the strength reduction factor as ϕ_{st} with the strut efficiency factor as β_s. Section 7.2.2 of AS3600-2009 states that for prismatic struts, β_s shall be taken as 1 when it is used to determine the design strength. For fan- and bottle-shaped compression fields that are unconfined, the β_s shall be taken as in Equation (1):

$$\beta_s = \frac{1}{1.0 + 0.66 cot^2\,\theta} \qquad \text{(within the limits } 0.3 \leq \beta_s \leq 1.0\text{)} \qquad (1)$$

where the angle (θ) is measured between the axis of the strut and the axis of a tie as illustrated in Figure 1. Note that when determining the ultimate strength of the steel tie the β_s is ignored. Note also that Section 7.2.1 of AS3600-2009 and Schlaich and Schäfer (1991) stated the three major geometrical classes of struts are typically of prismatic, fan or bottle shape, depending on the geometry of the compression field. Prismatic struts are the most basic type of strut.

Illustrated in Figure 1 is the horizontal force, N^*, which acts on the top surface of the bearing pad. This force is due to movement, shrinkage, temperature and prestress. The AS3600-2001 (Clause 12.1.2.6) states that the horizontal forces and movements from the supported members need to be considered when designing corbels. AS3600-2009 (Clause 12.3) repeats this statement and in addition it specifies that horizontal forces cannot be less than 20% of the vertical force. The American ACI 318-11 (Clause 11.8) requires that a section at face of support shall be designed to resist simultaneously a vertical force, a factor moment, and a factored horizontal tensile force.

The line of action of the vertical design load, V^*, acts at a distance 'a' from the column or wall face, this is illustrated in Figure 1. Clause 12.1.2.6 of AS3600-2001 states that "the line of action of the load may be taken at the outside edge of the bearing pad if any, or at the commencement of any edge chamfer, or at the outside face of the nib as appropriate". AS3600-2009 (Clause 12.3) repeats the aforementioned but then adds that this is for continuous nibs and also that the line of action should be "at one third the width of the bearing from the free end for a corbel". This standard then further stipulates that "where no bearing pad is provided, the line of action may be taken at the commencement of any edge chamfer, or at the outside face of the nib or corbel as appropriate".

DESIGN CHART

In line with the design philosophy of AS3600-2009, the strength of struts and ties should be considered in ultimate limit state. From Section 7.3.2 the ultimate strength of the steel tie, T, is as follows:

$$T = 0.8A_{st}f_{sy} \tag{2}$$

where A_{st} is the area of tension reinforcement and, f_{sy}, is the yield strength of reinforcement (see Section 3.2.1). The design strength of concrete strut, C, is determined as follows:

$$C = 0.6\beta_s 0.9 f'_c b_c d_c \tag{3}$$

where f'_c is the characteristic compressive (cylinder) strength of concrete at 28 days, b_c is the width of the corbel and, d_c is the width of the compression strut. Note that AS3600-2001 specified a value of 0.7 for both the tensile and compressive strength reduction factors. Using Figure 1 the vertical design load, V^*, and T can be expressed as:

$$V^* = C\sin\theta \tag{4}$$

$$T = C\cos\theta \tag{5}$$

Combining (4) and (5)

$$V^* = 0.54\beta_s f'_c b_c d_c sin\theta \tag{6}$$

From Figure 1, $d_c = x\cos\theta$ and introducing the factor, $n = \frac{x}{d}$, then Equation (6) can be written as:

$$V^* = 0.54\beta_s f'_c b_c n d cos\theta sin\theta \tag{7}$$

From the geometry of Figure 1, realising $x = nd$ and introducing the factor $\alpha = \dfrac{a}{d}$:

$$tan\theta = \frac{1-0.5n}{\alpha} \tag{8}$$

$$cos\theta = \frac{\alpha}{\sqrt{(1-0.5n)^2+\alpha^2}} \tag{9}$$

$$sin\theta = \frac{1-0.5n}{\sqrt{(1-0.5n)^2+\alpha^2}} \tag{10}$$

Combining (7), (9) and (10) and introducing the parameter, $K = \frac{V^*}{0.54\beta_s f'_c b_c d}$ gives:

$$K = n\alpha\frac{1-0.5n}{(1-0.5n)^2+\alpha^2} \tag{11}$$

Solving (11) for n gives:

$$n = \frac{(K+\alpha) - \sqrt{(K+\alpha)^2 - 4K(0.25K + 0.5\alpha)(1+\alpha^2)}}{2(0.25K + 0.5\alpha)} \tag{12}$$

Now combining (2), (3) and (5) and realising $d_c = x\cos\theta$ and $x = nd$, combining (8) and (12) and dividing throughout by $0.54\beta_s f'_c b_c d_c$ and introducing the parameter, provides:

$$0.8A_{st}f_{sy} = 0.54\beta_s f'_c b_c n d cos^2\theta \tag{13}$$

Combining (9) and (13) and introducing the parameter $R = \frac{0.8A_{st}f_{sy}}{0.54\beta_s f'_c b_c d}$

$$R = \frac{n\alpha^2}{(1-0.5n)^2 + \alpha^2} \tag{14}$$

The equations that enable the design chart (shown in Figure 1) to be plotted have been derived. For a particular value of K and α, n can be determined from (12). Using n and α (14) can be solved and the quantity of tension reinforcement determined from Figure 1.

Design Chart Example

Given the following parameters use the design chart and the associated constraints to design a corbel: V^* = 300kN, f'_c = 90MPa, a = 250mm, N^* = 5kN, f_{sy} = 500MPa, b_c = 150mm.

Step 1. Proportion corbel. $V^* < 0.2f'_c b_c d$ or $5.5b_c d$, hence, $300\times10^3 < 5.5\times150d$, hence, $d > 363.6$mm (Assume 35mm cover, hence D = 400mm and d = 365mm)

Step 2. Calculate K. $K = \frac{V^*}{0.54\beta_s f'_c b_c d}$ $\beta_s = \frac{1}{1.0+0.66cot^2\theta} = 0.77$, $\alpha = \frac{a}{d}$ = 0.685, $\therefore K = 0.1461$

Step 3. Determine R from Chart (Figure 3). Enter the chart on the vertical axis at a value of K = 0.155, project a line horizontally until it strikes the α=0.685 curve. Interpolation is permitted. Project down to the horizontal axis and read off the R value (see dotted line in Figure 1). $\therefore R = 0.1176$

Step 4. Calculate A_{st}. $R = \frac{0.8A_{st}f_{sy}}{0.6b_c d f_{c.cal}}$, hence A_{st} = 595.99mm^2

Step 5. Additional steel requirements. $N^* \geq 0.2V^*$, hence, N^* = 60kN. Extra steel must be provided for this load. $A_n = \frac{N^*}{0.8f_{sy}}$ = 155.55mm^2 hence, total main steel $A_s = A_{st} + A_n$ = 752 mm^2 .An additional area of steel equal to half of the main reinforcement should be distributed in the upper two thirds of the corbel. $\therefore A'_s$ = 375.78mm

Step 6. Check Shear Friction. $V^* < 0.8\mu A'_s f_{sy}$, where μ is the coefficient of friction and equals 1.4 for monolithic construction, $\therefore A'_s$ = 555.55mm^2

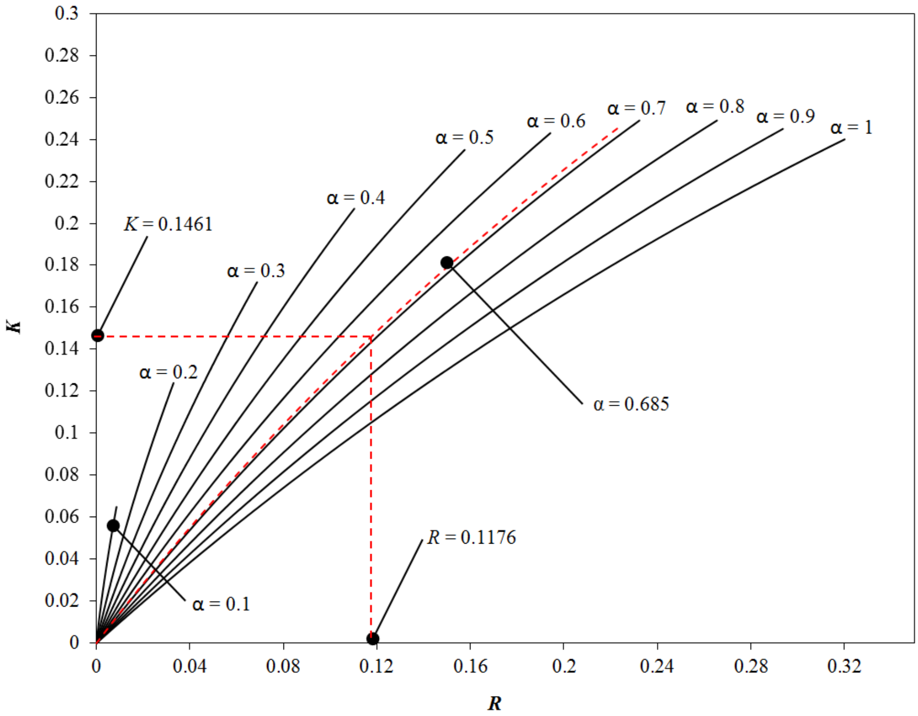

Figure 2. Corbel design chart 20-100MPa concrete

Verification of Design Chart

In order to verify the design chart a number of experimental failure loads determined from previous research was used for comparison with the theoretical failure load determined using the design chart method outlined in the aforementioned section. The chart allows only for tension failure, therefore the test data chosen for verification is limited to yielding of tension steel. The verification involves comparing the experimental failure load V_{exp} (kN) to the theoretical load capacity V_u (kN) as given by the design chart in Figure 2 excluding the use of strength reduction factors because it is a tensile failure.

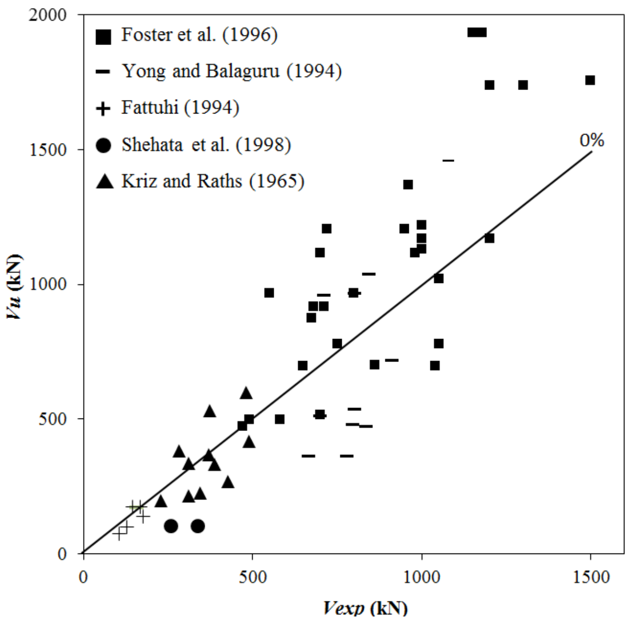

Figure 3. Comparison of experimental failure load V_{exp} to the theoretical load capacity V_u

Comparing the theoretical load capacity V_u with V_{exp} showed an average V_u/V_{exp} ratio of 1.24 for Foster et al. (1996). For Yong and Balaguru (1994b) the V_u/V_{exp} ratio was found to average 1.03 with the failure mode being beam shear. For Fattuhi (1994) a ratio of 0.94 was found indicating that our theoretical V_u was marginally lower than the experimental findings. Only two experimental V_{exp} was compared for Shehata et al. (1998) and V_u was significantly below the experimental value thus giving an average ratio of 0.34. A more favourable ratio of 0.96 was found for the experiment of Kriz and Raths (1965). Overall, Figure 3 shows that the majority of experimental failure loads vary by +/- 40% from the theoretical load (or line at 0%).

CONCLUSION

In this research a new corbel design chart is developed to aid the design process which eliminates the need for an iterative procedure. Relevant design constraints from AS3600-2001, AS3600-2009 and ACI318-11 design codes were highlighted and compared. Finally the design chart was verified through comparison with a number of experimental works. Future research may consider further validation using the Finite Element Method.

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